PARAMETER PLANE DESIGN OF COMPLEX ZERO COMPENSATORS

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THESIS

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by

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ABSTRACT

Control systems affected by mechanical or structural resonance contain at least one pair of complex poles in their corresponding transfer functions. The effect of the complex poles is to introduce resonance peaks which may cause instability in the closed-loop system. Such systems may be stabilized by the use of cascade compensators containing one pair of complex zeros.

The use of a parameter plane analysis, using a function of the location of the complex zeros as the two variables, in determining the correct location of the zeros of the compensator to guarantee stability and to ensure a desired transient response in the compensated system is illustrated.

As a result of the investigation of a number of systems affected by resonance phenomena using parameter plane techniques, design criteria for cascade complex zero compensators, based upon the interpretation of the 'stable region' of the parameter plane, are presented.

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I. INTRODUCTION

Structural resonances in mechanical systems which, for example, may be caused by the twisting of motor or load shafts can be modelled by the introduction of at least one pair of complex conjugate poles in the transfer function of the system model.

The effect of the complex poles is to introduce a resonant peak whose magnitude is dependent upon the damping ratio, ζ. The general form of an open loop transfer function containing a pair of complex poles may be written in the Bode form as:

$$G(j\omega) = \frac{K_x \prod_{i=0}^{m} (j\omega\tau_i + 1)}{(j\omega)^N \prod_{k=0}^{n} (j\omega\tau_k + 1)\left[\left(\frac{j\omega}{\omega_n}\right)^2 + j\left(\frac{2\zeta\omega}{\omega_n}\right) + 1\right]} \tag{1-1}$$

The magnitude of the pair of complex poles is:

$$\frac{1}{\left|\left(\frac{j\omega}{\omega_n}\right)^2 + j\left(\frac{2\zeta\omega}{\omega_n}\right) + 1\right|} = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} \tag{1-2}$$

whose maximum occurs where the ration of ω/ω_n is found to be

$$\omega/\omega_n = \sqrt{1 - 2\zeta^2} \quad \text{for} \quad \zeta < 0.707 \tag{1-3}$$

and the

$$\text{peak magnitude} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad \zeta < 0.707 \tag{1-4}$$

No resonance or peak value occurs for $\zeta > 0.707$.

A plot of the peak magnitude expressed in dB is shown in Fig. 1.1 which illustrates that resonance peaks with magnitudes as high as 50 dB may be encountered.

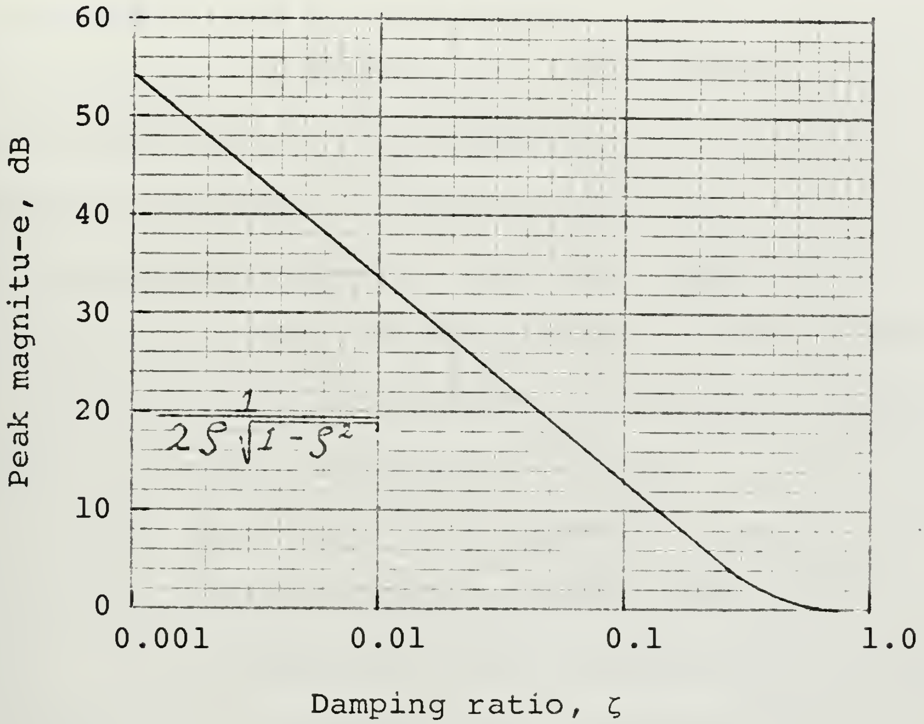


Figure 1.1

Peak magnitude as a function of damping ratio, ζ

If the system gain has to be maintained at a relatively high level in order to satisfy other criteria, the inherent resonance peaks of the complex poles may cause instability if the resonant frequencies are contained in the bandwidth of the system.

Stability may be ensured by the introduction of a cascade compensator whose effect is to cancel or reduce the resonant peaks produced by the complex poles. A very effective technique, if physically realizable, is to replace undesirable

poles with desirable ones by the introduction of a cascade compensator. Since the undesirable poles under present consideration are complex, an ideal solution would be obtained if the complex poles are cancelled exactly by complex zeros introduced by the compensator. Unfortunately, in practice, it is usually physically impossible to build this type of compensator because the exact location of the complex poles can not be predicted and in fact the complex poles may change position during the actual operation of the system.

The location of the complex zeros must, therefore, be chosen in a manner which will achieve:

1. stability of the system,
2. desired system response, and
3. be physically and economically realizable.

Lamdazuri [1] showed that a bounded region on the 'S' plane can be constructed with the property that, if the complex zeros are located within this region, stability can be assured. He arrived at this stability region by a method of trial and error using digital computer simulation. By further computer simulations of selected locations of the complex zeros within the stability region, he arrived at certain design criteria.

The intent of this thesis is to arrive at this stability region using parameter plane techniques and to obtain design criteria directly from the parameter plane data thus alleviating the need for trial and error simulation other than as a final check on specific system performance.

II. THE PARAMETER PLANE

The values of the parameters of a dynamic system determine its response to various stimuli since the parameter values specify the roots of the characteristic equation of the system.

The first parameter plane diagram was constructed by Vishnegradsky showing the dynamic operating characteristics as functions of the coefficients of the characteristic equation. In 1959, Mitrovic [2] introduced a method for determining the roots of the characteristic equation when the coefficients of the two lowest ordered terms vary independently. This method was later extended and generalized by Siljak [3,4] to provide present day parameter plane theory.

In general, parameter plane methods establish a relationship between the values of two chosen parameters, α and β, and the characteristic equation of a linear dynamic system by transforming the characteristic equation and using the fact that the transform variable, s, is complex, requiring that the real and imaginary parts of the characteristic equation go to zero independently. Two equations are thus determined whose simultaneous solution provides values for the parameters α and β expressed as functions of the roots of the characteristic equation.

A number of digital computer programs have been developed which solve the simultaneous equations and provide a graphical

relationship between the parameters and the roots of the characteristic equation in the form of curves on the α and β plane. When chosen 's' plane contours are mapped onto the α, β plane a variety of analysis and design problems can be solved as a function of the two specified parameters.

A most immediate and useful application of the parameter plane method is the study of stability. By mapping the imaginary axis of the 's' plane onto the parameter plane, stability regions can be defined. It is this property which will be used extensively in the investigation to follow.

Although the parameter plane displays the relationship between the roots of the characteristic equation and the two parameters of the system from which an acceptable root pattern may be selected, it does not indicate other performance characteristics of the system. To ensure that at least one performance specification is satisfied for all values of α and β, it may be imbedded in the characteristic equation by direct substitution and algebraic manipulation. The parameter plane based on the imbedded form of the characteristic equation will then guarantee that the desired performance specification is met for all values of α and β. The desired performance specification adopted in this investigation is that the error coefficient remains unchanged by the introduction of a cascade compensator. Hence this specification will be imbedded in the characteristic equation prior to obtaining parameter plane curves.

III. CASCADE COMPLEX ZERO COMPENSATORS

A. 'S' PLANE "STABILITY CIRCLES" FROM PARAMETER PLANE DATA

A system subject to mechanical resonances may become unstable if the system gain has to be maintained at a relatively high level. Landazuri [1] has shown that cascade compensators may be used to stabilize the system. Ideal compensation is achieved when the complex zeros of the compensator are superimposed on the complex poles of the plant resulting in exact cancellation of the effect of the resonances produced by the complex poles. Adequate compensation can, however, be achieved if the complex zeros are located in a position other than coincident with the complex poles.

Landazuri arrived at a region of the 's' plane bounded by "circles of stability" with the property that the location of complex zeros within this region guaranteed stability of the system. However, a rather difficult procedure of trial and error using root locus methods for specific complex zero locations was used to determine the limit of stability at a specified loop gain. The use of parameter plane methods simplifies the procedure considerably.

To establish a basis for comparison, the basic system used by Landazuri will be reexamined. A block diagram representation of the system is given in Fig. 3.1, where

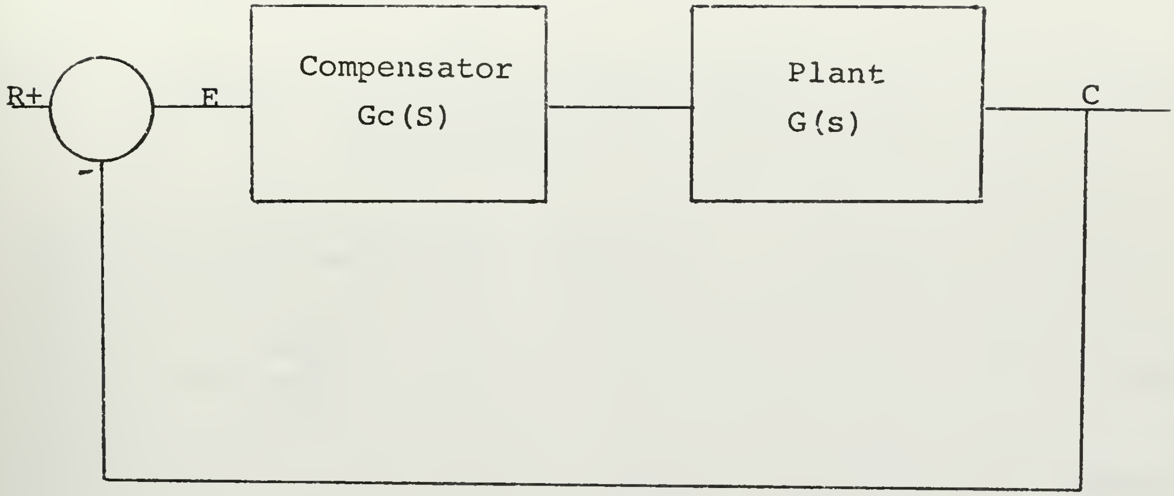


Figure 3.1

Block Diagram of Basic System

the transfer function of the plant and compensator is defined as:

$$G(s) \triangleq \frac{k}{s(s^2 + 0.1\ s + 1)} \qquad (3\text{-}1)$$

and

$$Gc(s) \triangleq \frac{s^2 + 2\zeta\omega_n\ s + \omega_n^2}{(s+10)(s+20)} \ . \qquad (3\text{-}2)$$

The real poles of the compensator were chosen such that their contribution to the transient response of the overall system will disappear before the maximum overshoot is reached.

The value of the gain, k, at the limit of stability was calculated to be 0.1. For this value of gain the error coefficient, Kv, is 0.1.

The system transfer function is

$$GcG(s) = \frac{k(s^2 + 2\zeta\omega_n s + \omega_n^2)}{s(s+10)(s+20)(s^2 + 0.1 s + 1)} \tag{3-3}$$

and the error coefficient, Kv, is

$$Kv = \frac{k\omega_n^2}{200} \quad . \tag{3-4}$$

If the error coefficient is imbedded, equation (3-3) becomes

$$GcG(s) = \frac{200Kv/\omega_n^2 \; s^2 + 2(200Kv)\;\zeta/\omega_n \; s + 200Kv}{s(s+10)(s+20)(s^2+0.1 s + 1)} \tag{3-5}$$

and defining $\alpha = 1/\omega_n^2$ and $\beta = \zeta/\omega_n$

allows equation (3-5) to be written as

$$GcG(s) = \frac{200Kv\ \alpha\ s^2 + 2(200Kv)\ \beta\ s + 200Kv}{s(s+10)(s+20)(s^2 + 0.1 s + 1)} \; . \tag{3-6}$$

The characteristic equation of the system:

$$1 + GcG(s) = 0 \tag{3-7}$$

will, therefore, contain the two parameters α and β.

The results of implementing equation (3-7) as a parameter plane problem for values of Kv 10, 50 and 100 percent above its value at the limit of stability of the uncompensated system are shown in Fig. 3.2, 3.3 and 3.4 respectively.

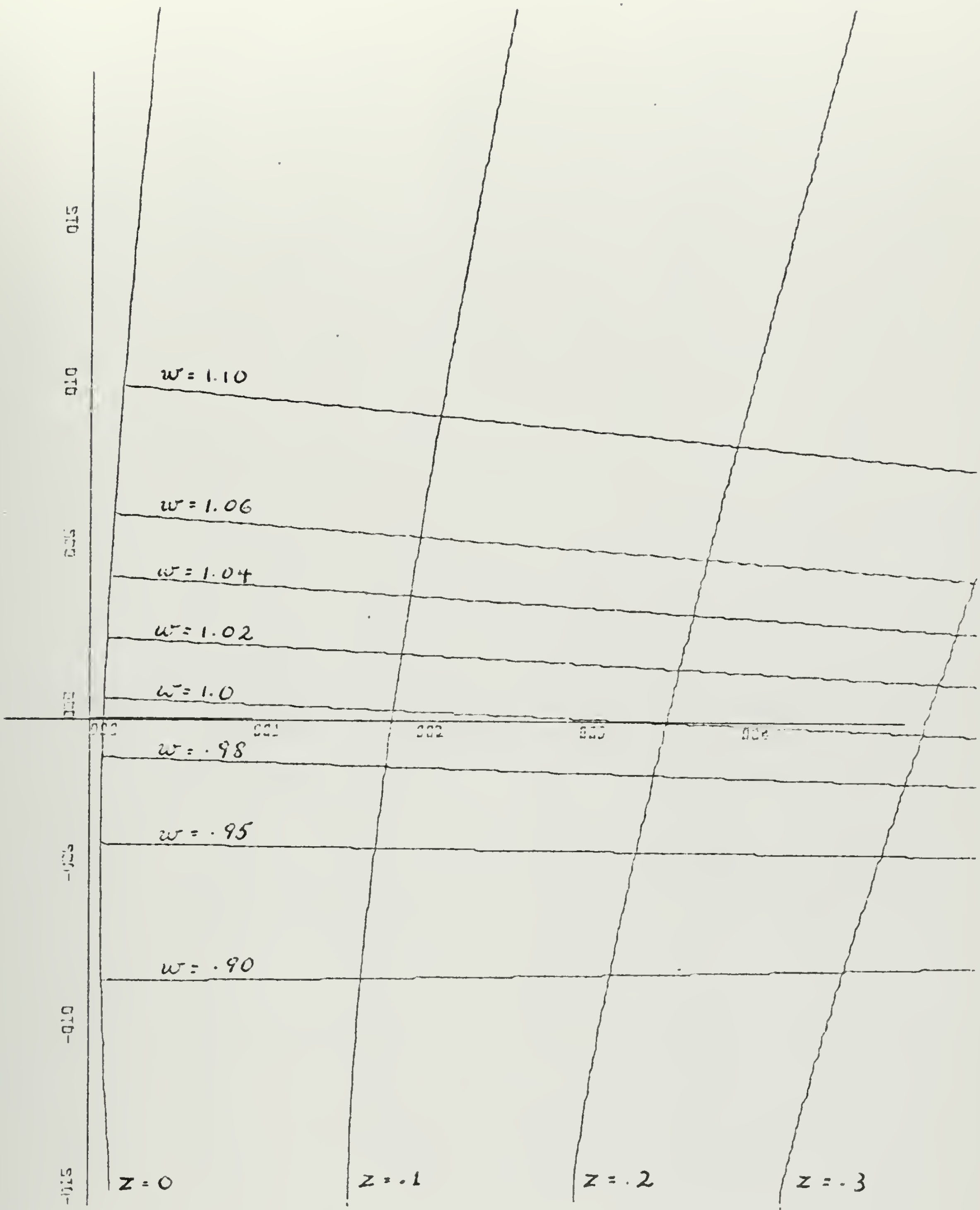
A mapping of the imaginary axis of the 's' plane is represented by the zeta = 0.0 curve on the parameter plane,

i.e., values of α and β chosen to lie on the zeta = 0.0 curve ensure that one pair of complex roots of the system is located on the imaginary axis of the 's' plane. The system is, therefore, at the limit of stability if all of the remaining roots are in the left-half plane. From Figs. 3.2, 3.3 and 3.4 values of α and β that lie on the respective zeta = 0.0 curves define, therefore, the stability boundary on the 's' plane in terms of the ω_n and ζ of the complex zeros since

$$\omega_n = \sqrt{1/\alpha} \qquad \text{and} \qquad \zeta = \beta\ \omega_n \quad . \tag{3-8}$$

The resultant stability boundaries are shown in Fig. 3.5 for three values of Kv. It should be noted that these boundaries are identical to the ones determined by Landazuri with the exception that they should not be referred to as stability circles since their actual shape deviates considerably from circular as illustrated particularly by the results obtained from Kv = 0.20.

The use of parameter plane data has considerably simplified the procedure of obtaining 's' plane stability boundaries. A considerable amount of labor remains, however, in converting parameter plane data for use on the 's' plane. This is, in fact, unnecessary since the parameter plane curves can be drawn in terms of the ω_n and ζ of the complex zeros directly. The resultant graphs impart as much, and in fact more, information as the 's' plane without intermediate steps as will be shown in the following section.

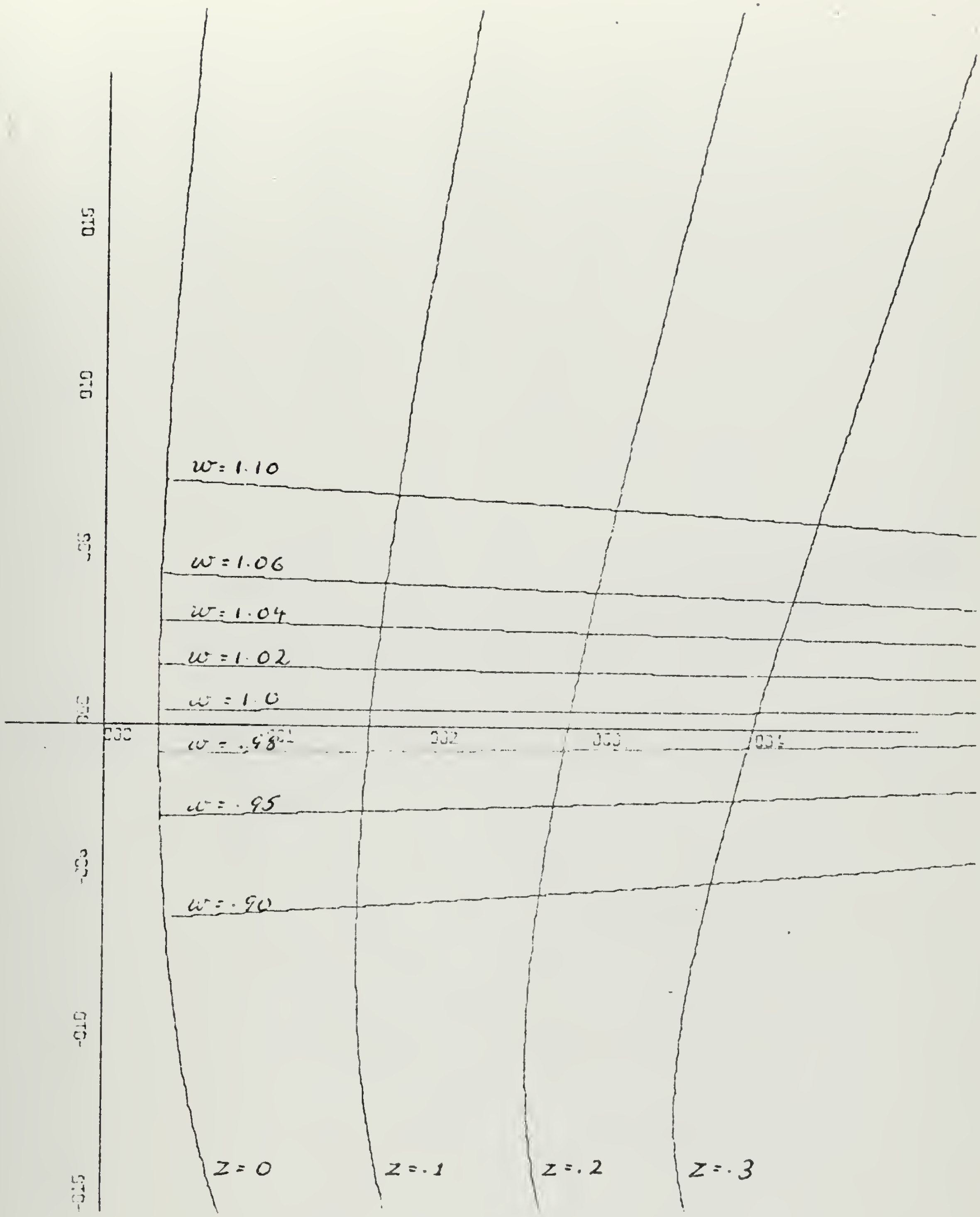


Scale: α = 1.0/in, β = 0.5/in

Figure 3.2

Parameter Plane Curves for $\alpha = 1/\omega_n^2$ and $\beta = \zeta/\omega_n$

Kv = 0.11

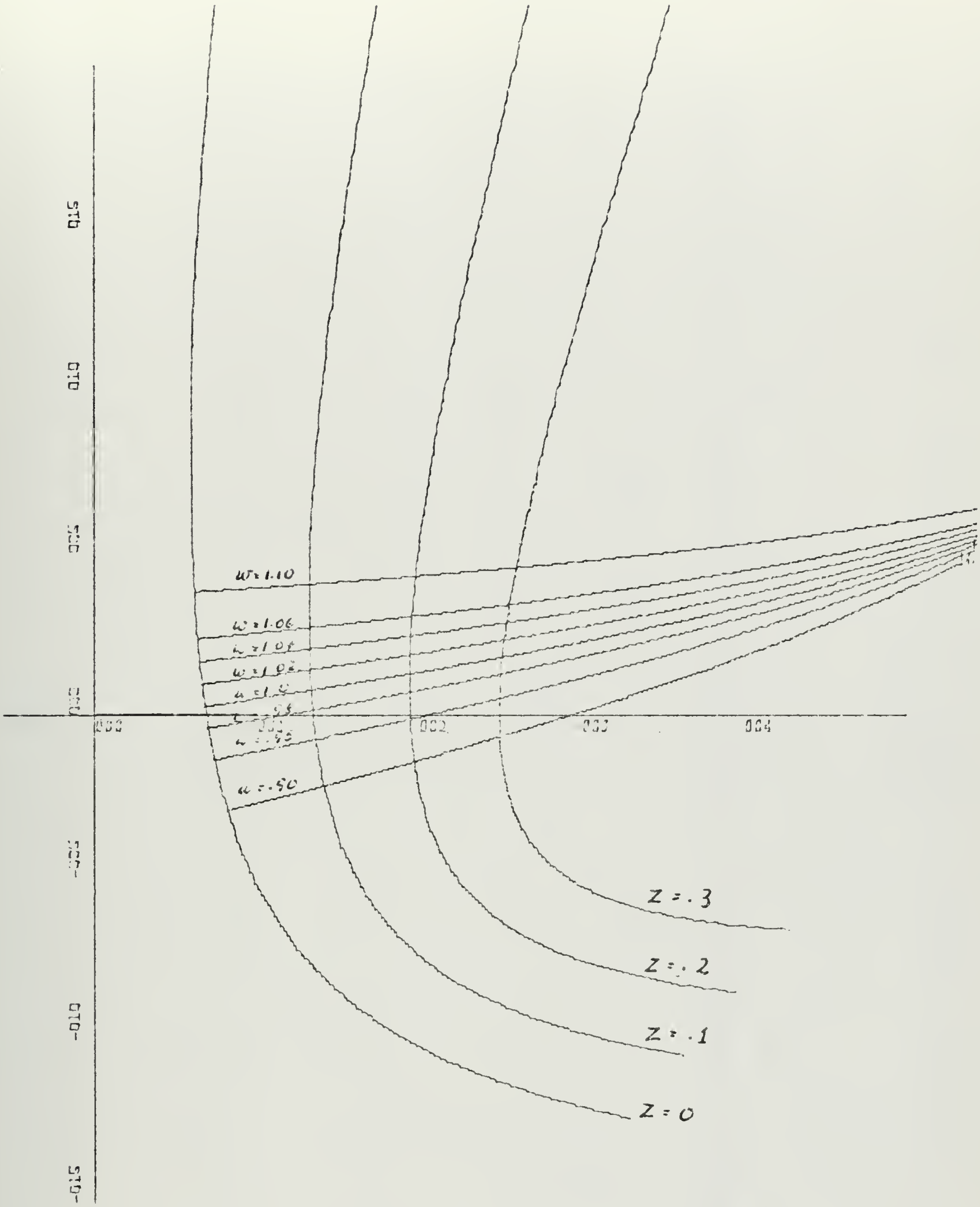


Scale: α = 1.0/in, β = 0.5/in

Figure 3.3

Parameter Plane Curves for $\alpha = 1/\omega_n^2$ and $\beta = \zeta/\omega_n$

Kv = 0.15



Scale: α = 1.0/in, β = 0.5/in

Figure 3.4
Parameter Plane Curves for $\alpha = 1/\omega_n^2$ and $\beta = \zeta/\omega_n$
Kv = 0.20

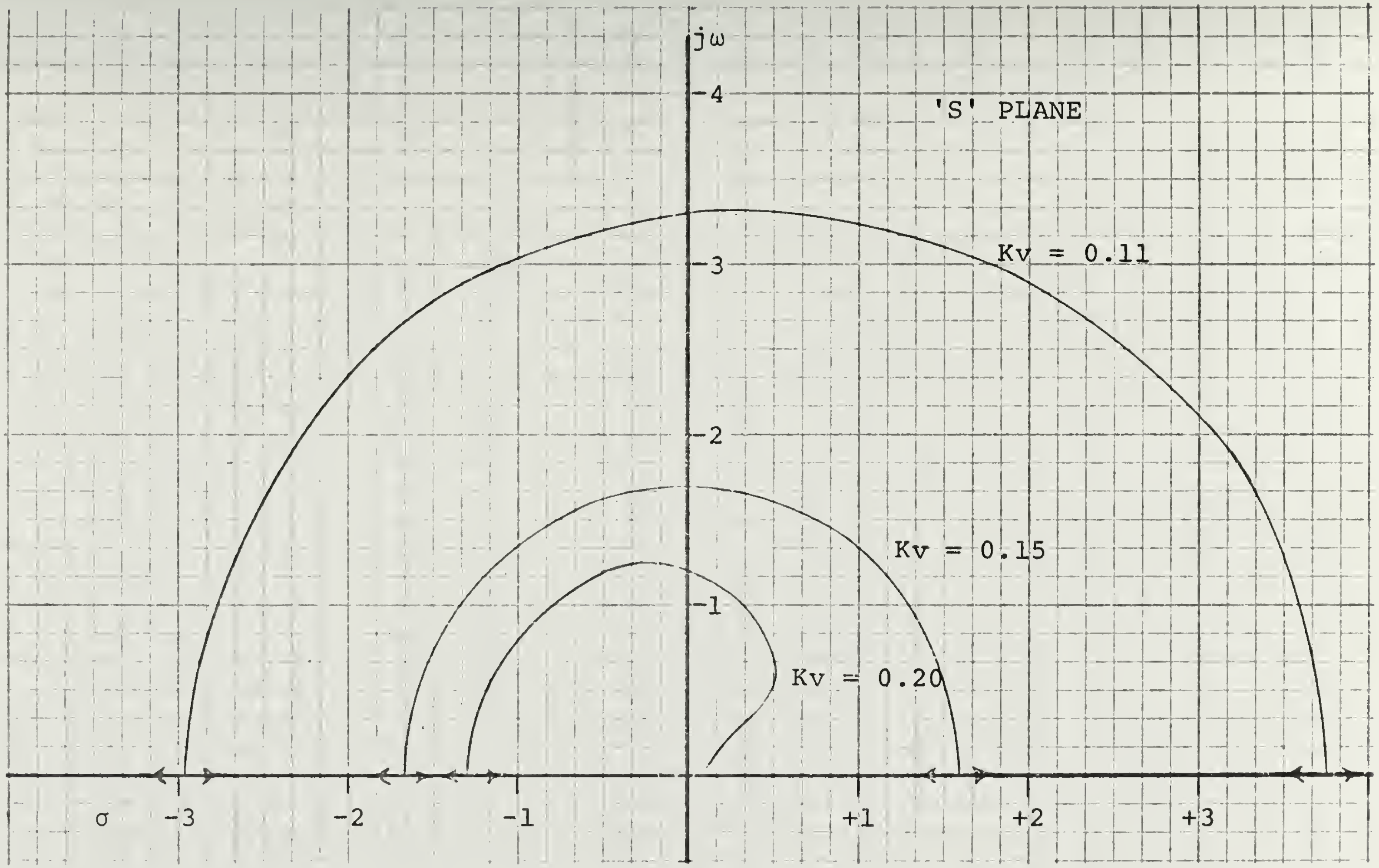


Figure 3.5

'S' Plane "Stability Regions" for three Values of Kv

B. STABILITY REGIONS ON THE PARAMETER PLANE

The stability boundaries as shown in Fig. 3.5 clearly define that region on the 's' plane in which a pair of complex zeros may be located to guarantee stability of the system. Since these boundaries were determined from parameter plane data, the parameter plane itself can be manipulated to show the stability region just as clearly as the 's' plane.

The two parameters α and β were previously defined as functions of the location of the complex zeros, namely $1/\omega_n^2$ and ζ/ω_n respectively. This choice was imposed by the fact that the error coefficient was imbedded in the transfer function of the system. Parameter plane curve calculations must, therefore, be based upon these two parameters. For plotting purposes, however, any function of the original α and β may be used. One obvious choice was to define two new parameters

$$\alpha_1 \triangleq \sqrt{1/\alpha} \qquad \text{and} \qquad \beta_1 \triangleq \beta\sqrt{1/\alpha} \tag{3-9}$$

in terms of α and β with the result that $\alpha_1 = \omega_n$ and $\beta_1 = \zeta$ of the complex zeros.

Figures 3.6, 3.7 and 3.8 were plotted with these new axis, but otherwise are identical to Figs. 3.2, 3.3 and 3.4 respectively. The stability region is clearly defined by the zeta = 0 curve indicating that values of α and β chosen to lie on this curve will guarantee that one pair of roots of the closed-loop system will lie on the imaginary axis of the 's' plane and, if the zeta = 0 curve forms an enclosed

region on the parameter plane, that the remaining roots are in the left half of the 's' plane.

Constant zeta curves of values other than zero can, of course, also be plotted as shown. This means that values of α and β chosen to lie on these constant zeta curves will guarantee that one pair of roots of the closed-loop system will have a corresponding zeta and, if the constant zeta curve lies within the stability region, that the system is stable; i.e., all roots are in the left half of the 's' plane. The constant zeta curves can, therefore, be used as a tool in meeting design specifications since they define values of α and β which will result in a specified zeta. Unfortunately they do not guarantee dominance of the selected roots and should, therefore, only be used as a rough guide and not as an absolute indication of the closed-loop system performance. The problem of dominance of selected roots will be further discussed in sections to follow.

To illustrate, however, the information available from the constant zeta curves, two sets of values of ω_n and ζ were chosen as indicated by points A and B on Fig. 3.7. Clearly the system response for values of ω_n and ζ defined by point B should be more damped than the response for values of ω_n and ζ defined by point A. That this is, in fact, the case is illustrated in Fig. 3.9 and 3.10 which represent the transient response of the system to a step input for values of ω_n and ζ defined by points A and B respectively.

Since the stability region exists for values of $|\zeta| > 1$ as well, it is immediately obvious that the system can be stabilized not only by the addition of a pair of complex zeros, but also by two real zeros, which was not obvious from the 's' plane stability region.

For Kv = 0.15, values of $\omega_n = 0.5$ and $\zeta = 4.0$ were chosen which result in the location of two real zeros at s = -0.063 and s = -3.94 as shown in Fig. 3.12. That the resultant system is stable is immediately obvious from Fig. 3.12 and illustrated in Fig. 3.11 which represents the transient response of the system for the chosen values of ω_n and ζ.

Since it is considerably simpler and more economical to build real zero compensators than complex ones, the practical designer would certainly prefer their use if possible. The parameter plane, therefore, allows him to determine whether real zero compensators can be used for a particular system.

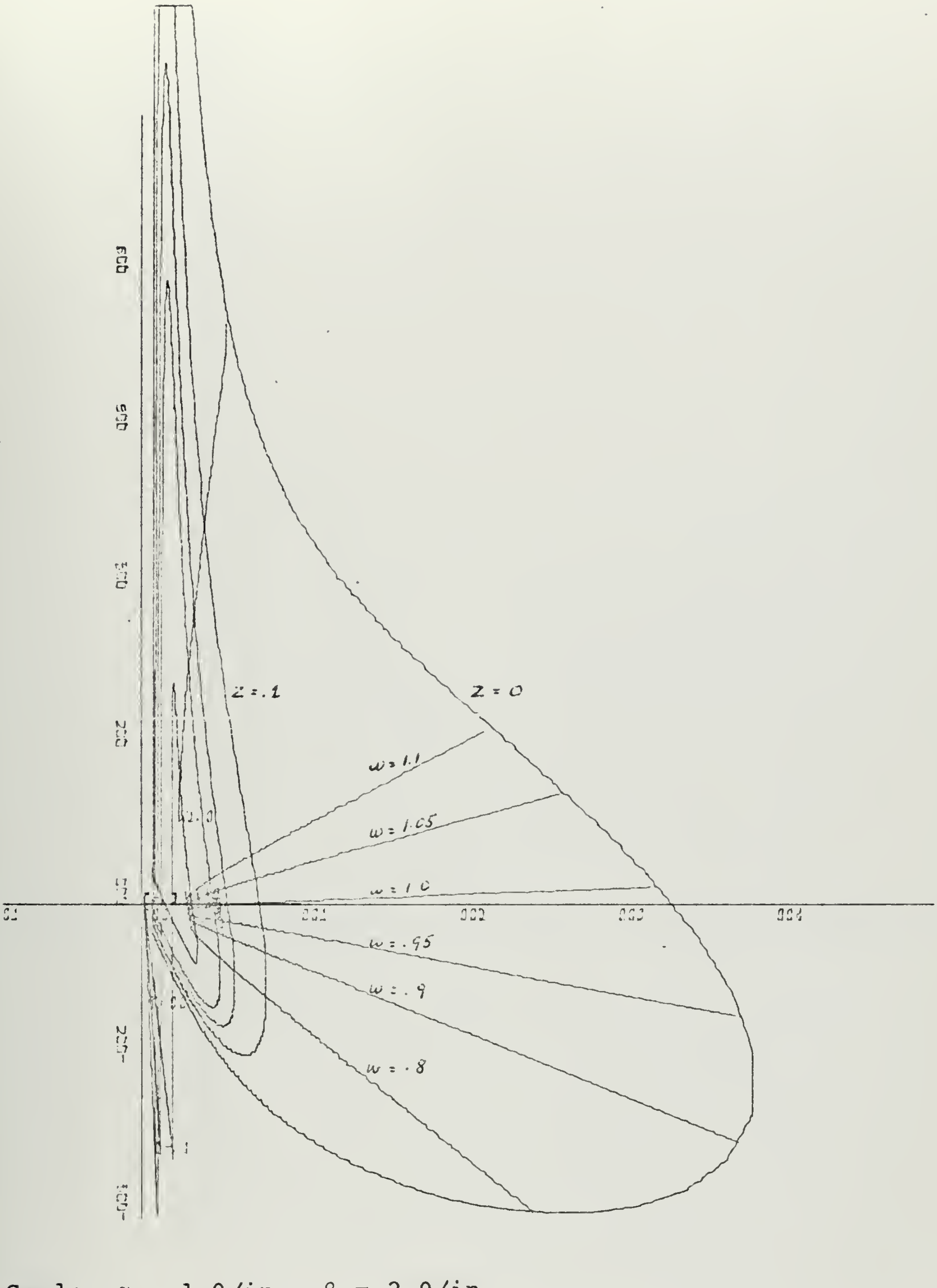
Landazuri [1] arrived at design criteria which stated that the complex zeros should be located so that:

1. $\zeta_z => \zeta_p$, and

2. $\omega_z =< \omega_p$.

The parameter plane curves do not directly indicate criteria 1., but criteria 2. is obvious from the curves. Since the values of ω_n for curves of constant zeta ≥ 0.1 are all less than 1.0 which is the value of ω_p, a value of $\omega_n < \omega_p$ will result in a more damped transient response.

It has been shown that the parameter plane can display stability regions defined by the zeta = 0 curve. The

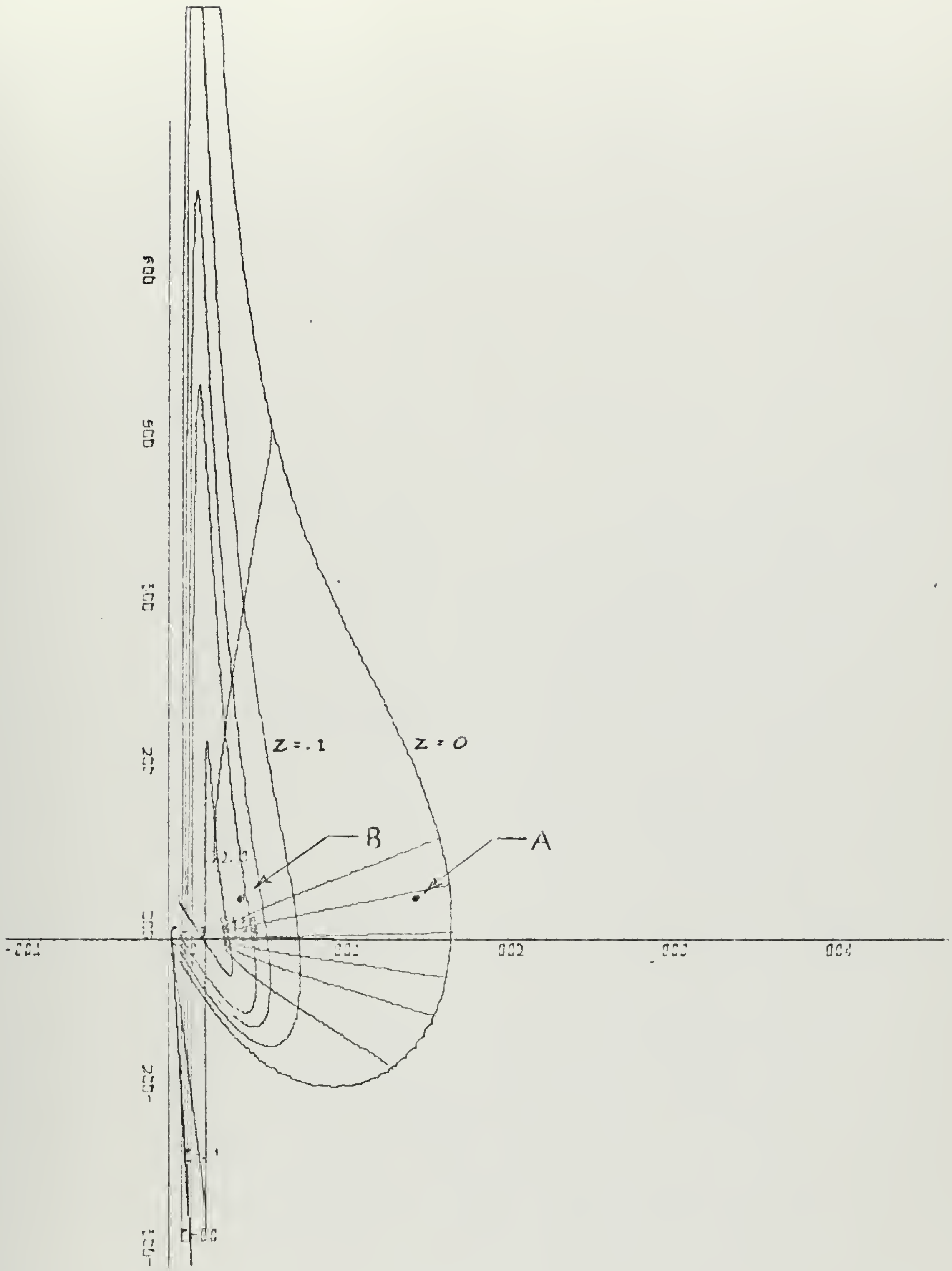


Scale: α = 1.0/in, β = 2.0/in

Figure 3.6

Parameter Plane Curves for $\alpha = \omega_n$ and $\beta = \zeta$

Kv = 0.11

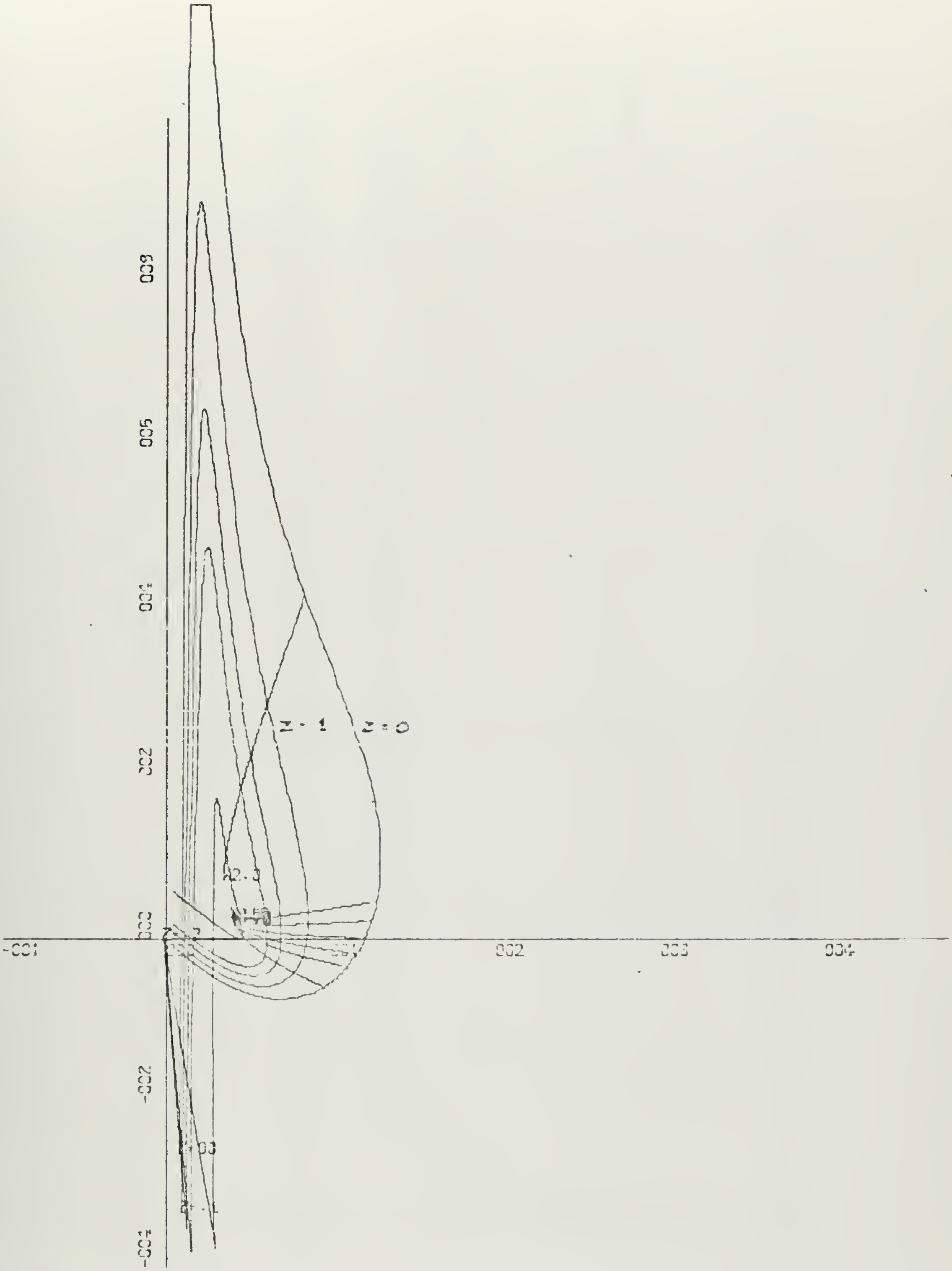


Scale : α = 1.0/in, β = 2.0/in

Figure 3.7

Parameter Plane Curves for $\alpha = \omega_n$ and $\beta = \zeta$

Kv = 0.15

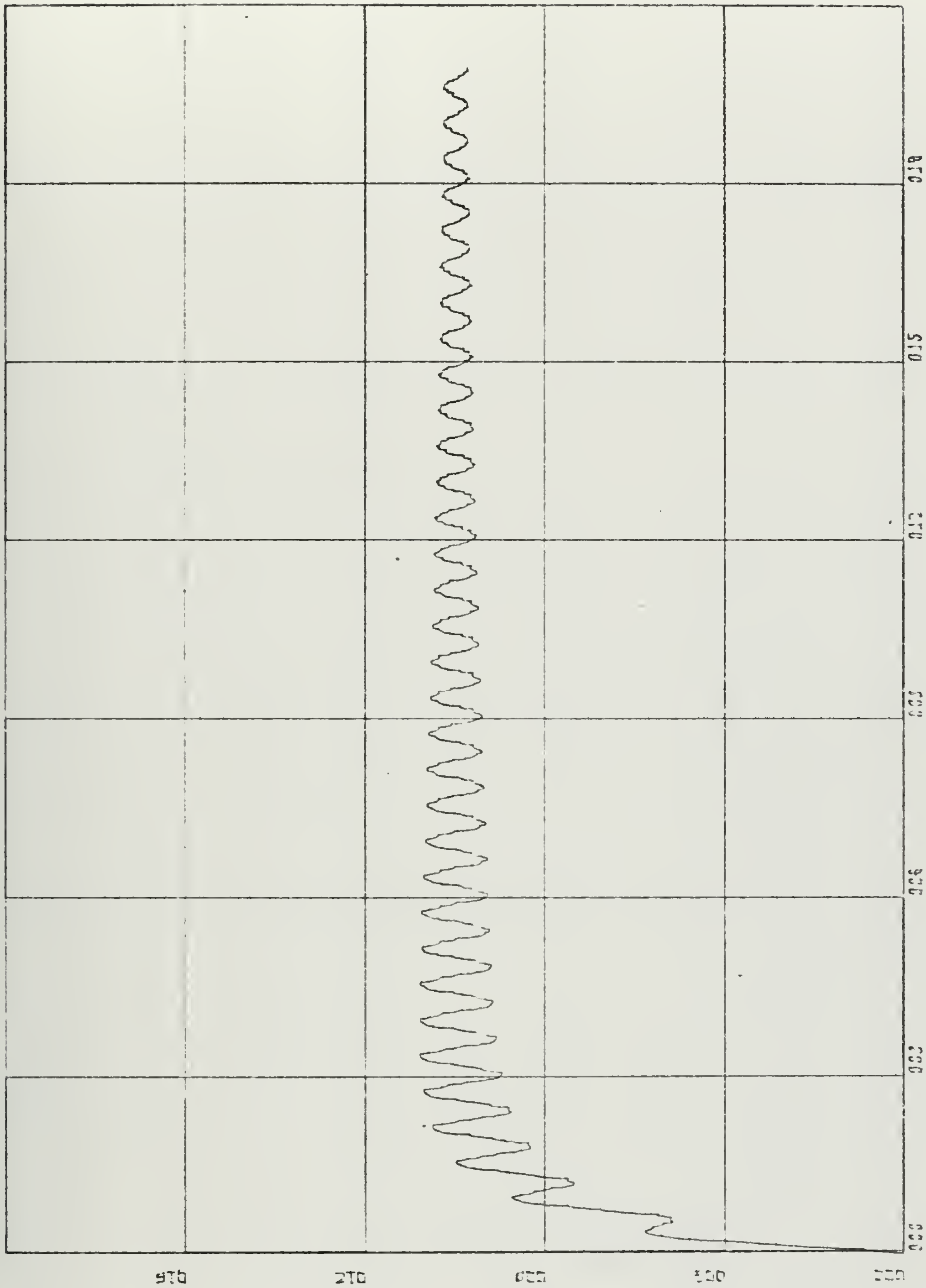


Scale α = 1.0/in, β = 2.0/in

Figure 3.8

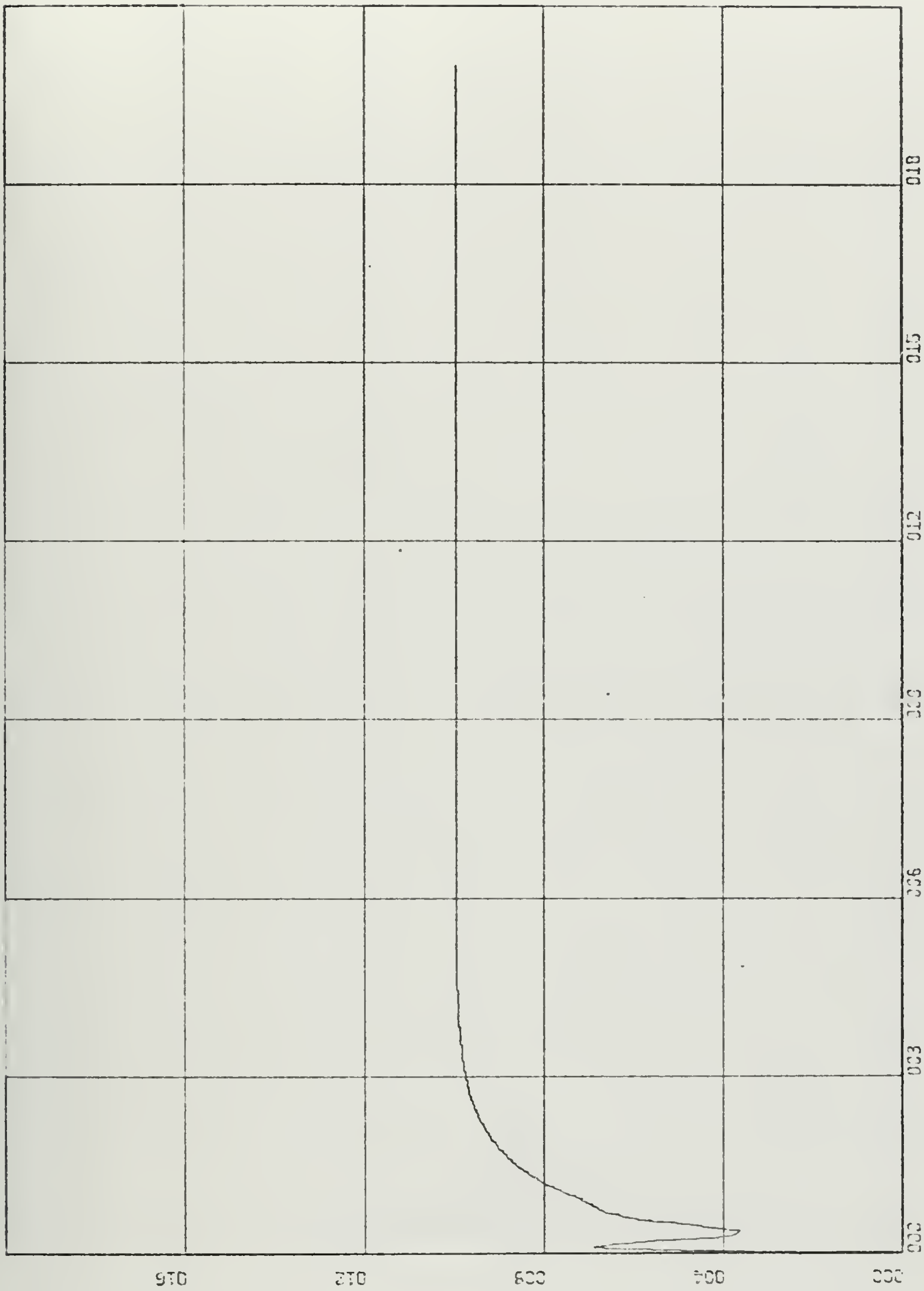
Parameter Plane Curves for $\alpha = \omega_n$ and $\beta = \zeta$

Kv = 0.20



Scale x = 30 sec./in., y = .4 units/in.

Figure 3.9
Transient Response for Point A.



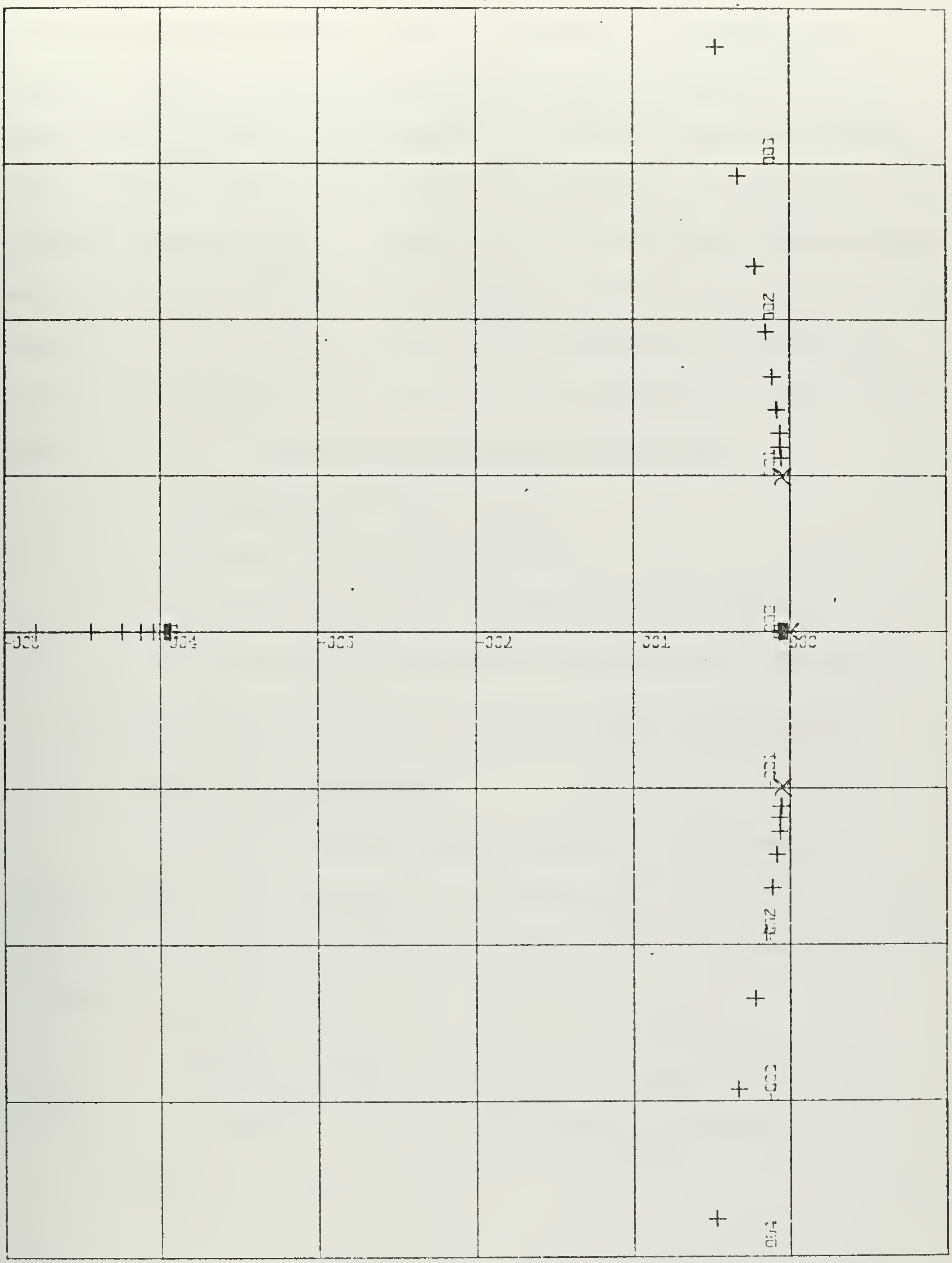
Scale: x = 30 sec./in, y = .4 units/in

Figure 3.10
Transient Response for Point B.

Scale: x = 30. sec./in, y = .4 units/in

Figure 3.11

Transient Response for Compensated System with Real Zeros.



Scale: x and y = 1.0/in

Figure 3.12

Root Locus for Compensated System with Real Zeros.

parameter plane can also give guidance in locating the complex zeros to ensure a specified type of response by using the constant zeta curves. It would appear, therefore, that the parameter plane is ideally suited as an aide to the designer faced with the problem of complex zero compensation. So far, however, only a very simple system has been investigated. The following section will, therefore, generalize the problem of cascade complex zero compensators and investigate a number of more complicated cases such as:

1. systems of higher order,
2. effect of round-off poles,
3. systems with two resonance peaks, and
4. systems with resonance-antiresonance doublets.

C. PARAMETER PLANE DESIGN OF COMPLEX ZERO COMPENSATORS

1. The General Equation

The block diagram representation of the general system under consideration is illustrated in Fig. 3.1. The generalized transfer function of the system may be expressed in the following form

$$GcG(s) = \frac{k(s^2+2\zeta\omega_n s+\omega_n^2)}{(s+p)^2} \cdot \frac{K \prod_{i=0}^{\ell}(s+z_i) \prod_{i=0}^{m}[s^2+2\eta_i\mu_i+\mu_i^2]}{s^N \prod_{k=0}^{n}(s+p_k) \prod_{k=0}^{q}[s^2+2\eta_k\mu_k+\mu_k^2]} \tag{3-10}$$

where: a. K is large enough to cause instability in the uncompensated system,

b. z_i, p_k, η_i, μ_i, η_k and μ_k are known,

c. p is chosen sufficiently large so that its contribution to the transient response of the compensated system is negligible,

d. $k = \frac{p^2}{\omega_n^2}$ so that the error coefficient, Kv, remains unchanged, and

e. η and μ are equivalent to the generally accepted interpretation of ζ and ω_n respectively and are used only to differentiate the zeros of the compensator.

In order to transform equation (3-10) into an acceptable parameter plane form the two parameters α and β are defined as:

$$\alpha \triangleq 1/\omega_n^2 \qquad \text{and} \qquad \beta \triangleq \zeta/\omega_n$$

and upon substitution the general transfer function becomes:

$$GcG(s) = \frac{Kp^2(\alpha s^2+2\beta s+1)\prod_{i=0}^{\ell}(s+z_i)\prod_{i=0}^{m}(s^2+2\eta_i\mu_i+\mu_i^2)}{s^N\prod_{k=0}^{n}(s+p_i)(s+p)^2\prod_{k=0}^{q}(s^2+2\eta_k\mu_k+\mu_k^2)} \qquad (3\text{-}11)$$

from which the characteristic equation (3-7) can be formed.

To illustrate the use of the parameter plane in the design of a suitable complex zero compensator for a system affected by mechanical resonances, a number of examples will be studied which contain various combinations of pole and zero terms included in equation (3-11).

The mathematical solution for the parameter plane curves will, of course, allow values of α and β which may be both positive or negative. Physically, both positive and negative values of β are permissible since a positive β indicates a complex pair or two real zeros in the left half of the 's' plane, while a negative β indicates zeros in the right-half plane. Negative values of α, however, are not permitted since no corresponding physical interpretation exists. Negative values of α are, therefore, ignored in the interpretation of stability regions.

Whenever negative values of α occur in the parameter plane solution, the previously defined transformation (3-9) presents obvious difficulties. The problem can, of course, be resolved by setting $\alpha_1 = 0$ whenever α is negative or by taking the square root of the absolute value of $1/\alpha$. Digital computer plotting of parameter plane curves under these conditions introduces extraneous lines whenever α becomes negative so that the clarity of the parameter plane presentation diminishes considerably. For this reason the following parameter plane curves are plotted with $\alpha = 1/\omega_n^2$ and $\beta = \zeta/\omega_n$ without further transformation.

The stability regions are still as clearly defined as with the previously defined transformation, but negative values of α must be ignored, and it is not immediately obvious whether a chosen combination of α and β will result in a pair of complex zeros or in two real zeros. The advantage of clearly defined regions of stability obtained by plotting

α and β directly outweighs the indicated disadvantages and, hence, in the following parameter plane analysis α and β will be plotted without further transformation.

2. Analysis of a Fourth-Order System

The type 1 - 4th order system under investigation has an uncompensated transfer function

$$G(s) = \frac{K}{s(s+2)(s^2+0.1\ s+1)} \tag{3-12}$$

and its root locus is shown in Fig. 3.13.

The value of the gain, K, at the limit of stability was found to be 0.236 and the corresponding error coefficient, Kv, is 0.118. The error coefficient was increased by 50 percent to a value of 0.177 and the system was stabilized by the introduction of a cascade complex zero compensator whose transfer function is:

$$Gc(s) = \frac{100/\omega_n^{\ 2}\ (s^2+2\zeta\omega_n + \omega_n^{\ 2})}{(s+10)^2} \quad . \tag{3-13}$$

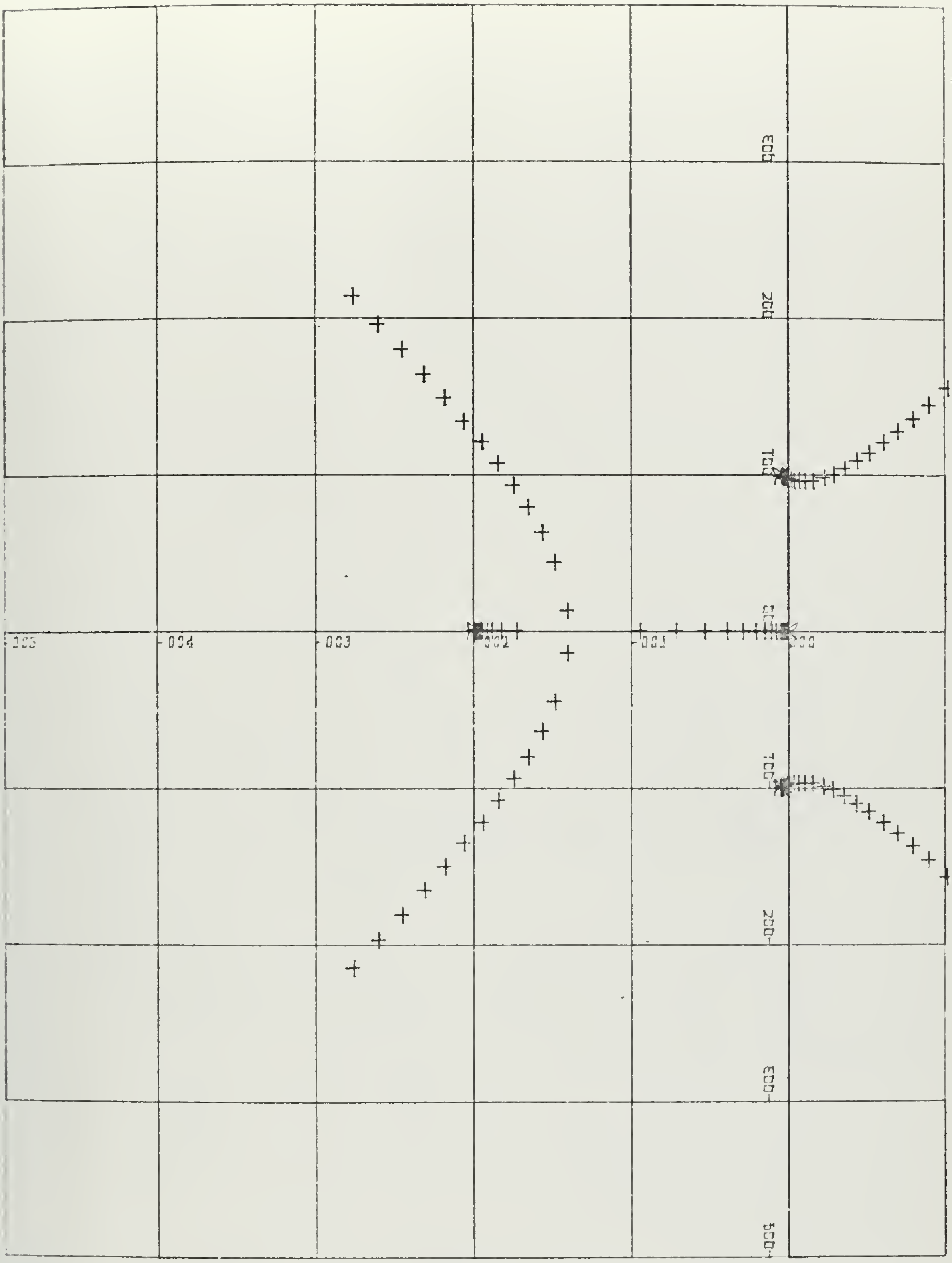
The resultant parameter plane curves are shown in Fig. 3.14 with the stability region clearly defined as the area bounded by the zeta = 0 curve. Bounded regions are also defined for zetas equal to 0.1, 0.2 and 0.5 which are entirely contained within the stability area. Thus values of α and β chosen to lie within these bounded regions would guarantee that all the roots of the closed-loop system would have a corresponding zeta or higher.

From the parameter plane, the values of $\alpha = 2.5$ and $\beta = -1.15$, which lie within the area bounded by zeta = 0.5, should give the system a transient response which is most acceptable from the standpoint of maximum overshoot and settling time. That this is, in fact, the case is shown in Fig. 3.15 which represents the transient response of the system for the indicated values of α and β. The corresponding root locus is shown in Fig. 3.16.

Two other values of α and β were chosen to establish a basis of comparison. The transient response for $\alpha = 1.0$ and $\beta = 0.0$ is shown in Fig. 3.17 and the corresponding root locus in Fig. 3.18. From Fig. 3.18, it is clear that almost exact cancellation of the complex poles has taken place which accounts for the highly damped response indicated in Fig. 3.17 for the chosen value of gain. This type of response was not directly obvious from the parameter plane and is due to the fact that almost exact cancellation has taken place with the resultant residues due to the complex poles being extremely small.

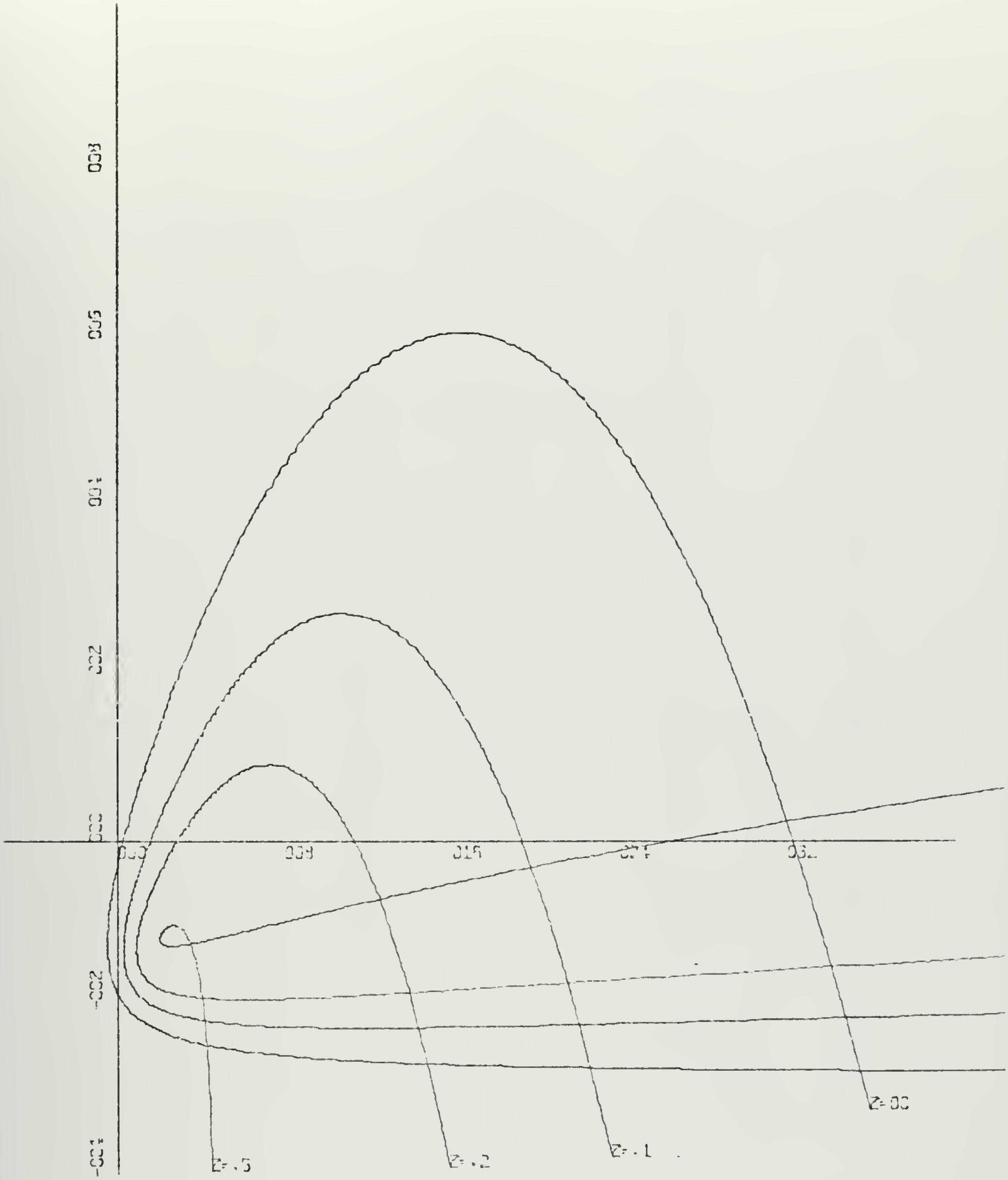
The oscillatory response as shown in Fig. 3.19 for $\alpha = 16.0$ and $\beta = 0.0$ was expected from the parameter plane data although its exact nature was not apparent since it is dependent upon the residue of the complex poles at the particular gain.

It was previously mentioned that, unfortunately, the parameter plane does not give an indication of dominance of the closed-loop system roots selected by a particular choice



Scale x and y = 1.0/in

Figure 3.13
Root Locus of Uncompensated 4th-order System.



Scale: α = 8.0/in, β = 2.0/in

Figure 3.14

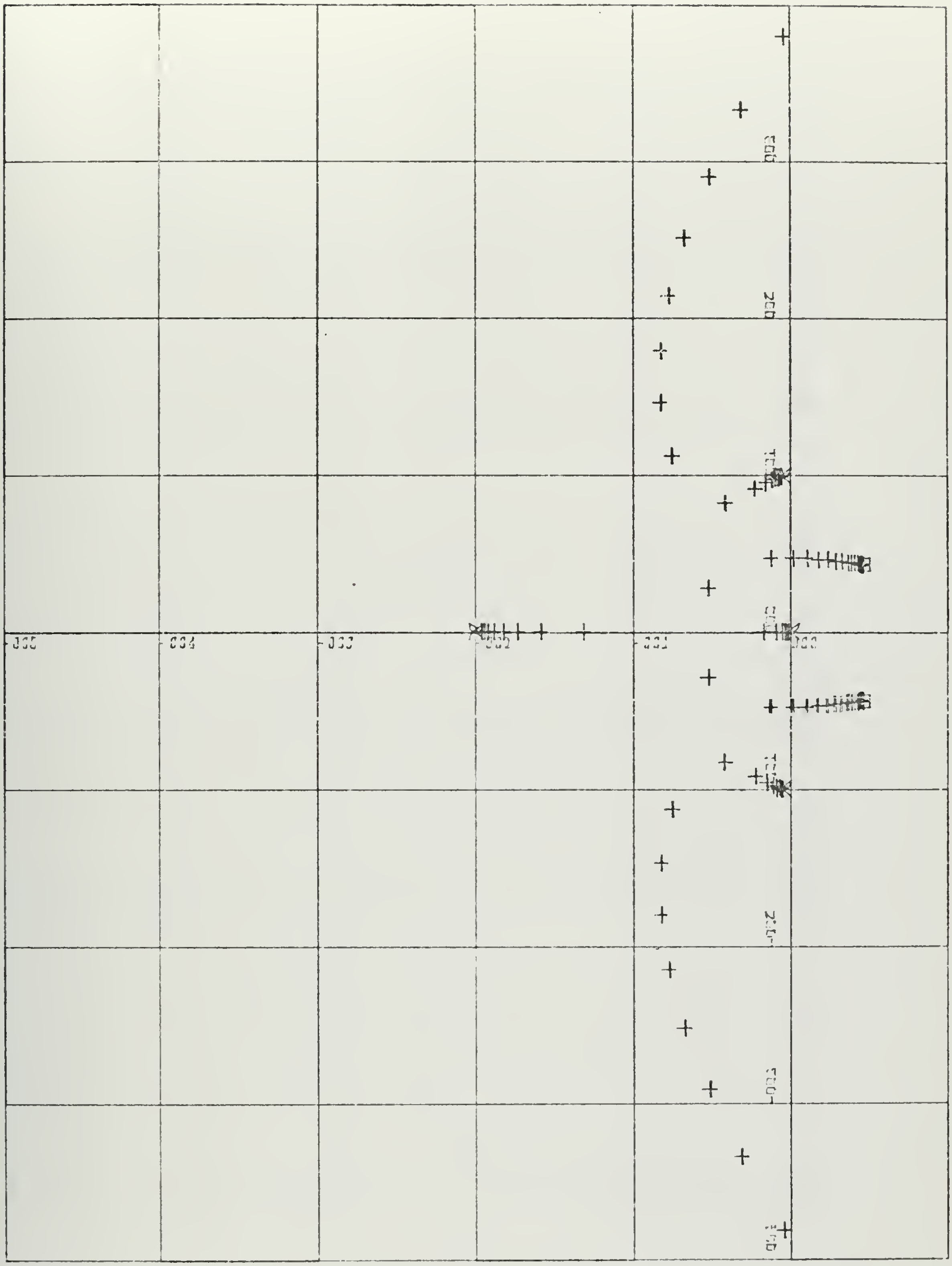
Parameter Plane Curves of 4th-order System.

Scale: x = 30 sec/in, y = .4 units/in

Figure 3.15

Transient Response of Compensated 4th-order System

$\alpha = 2.5$, $\beta = -1.15$



Scale: x and y = 1.0/in

Figure 3.16

Root Locus of Compensated 4th-order System

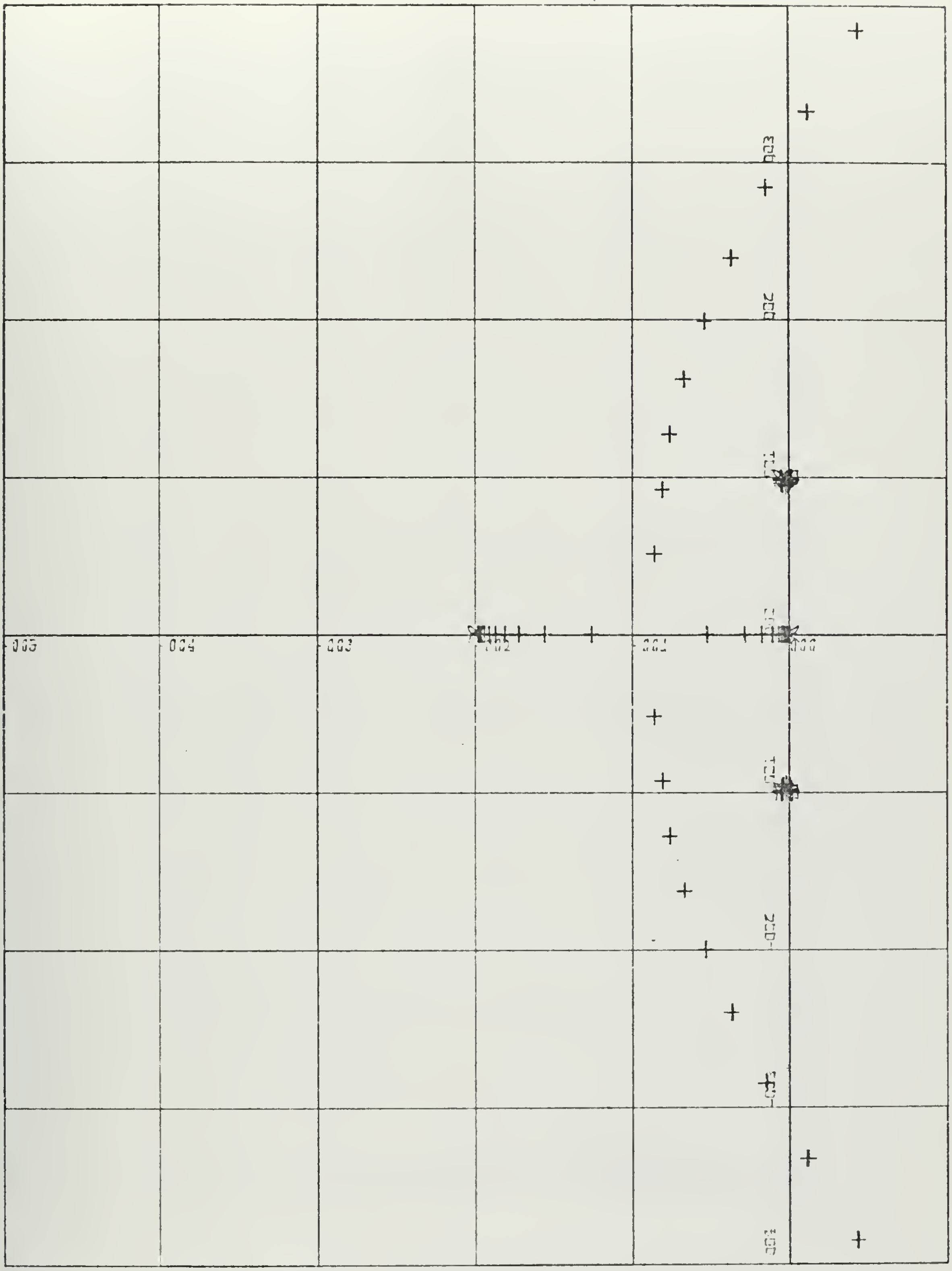
$\alpha = 2.5$, $\beta = -1.15$

Scale: x = 30. sec/in, y = 0.4 units/in

Figure 3.17

Transient Response of Compensated 4th-order System

$\alpha = 1.0$, $\beta = 0.0$

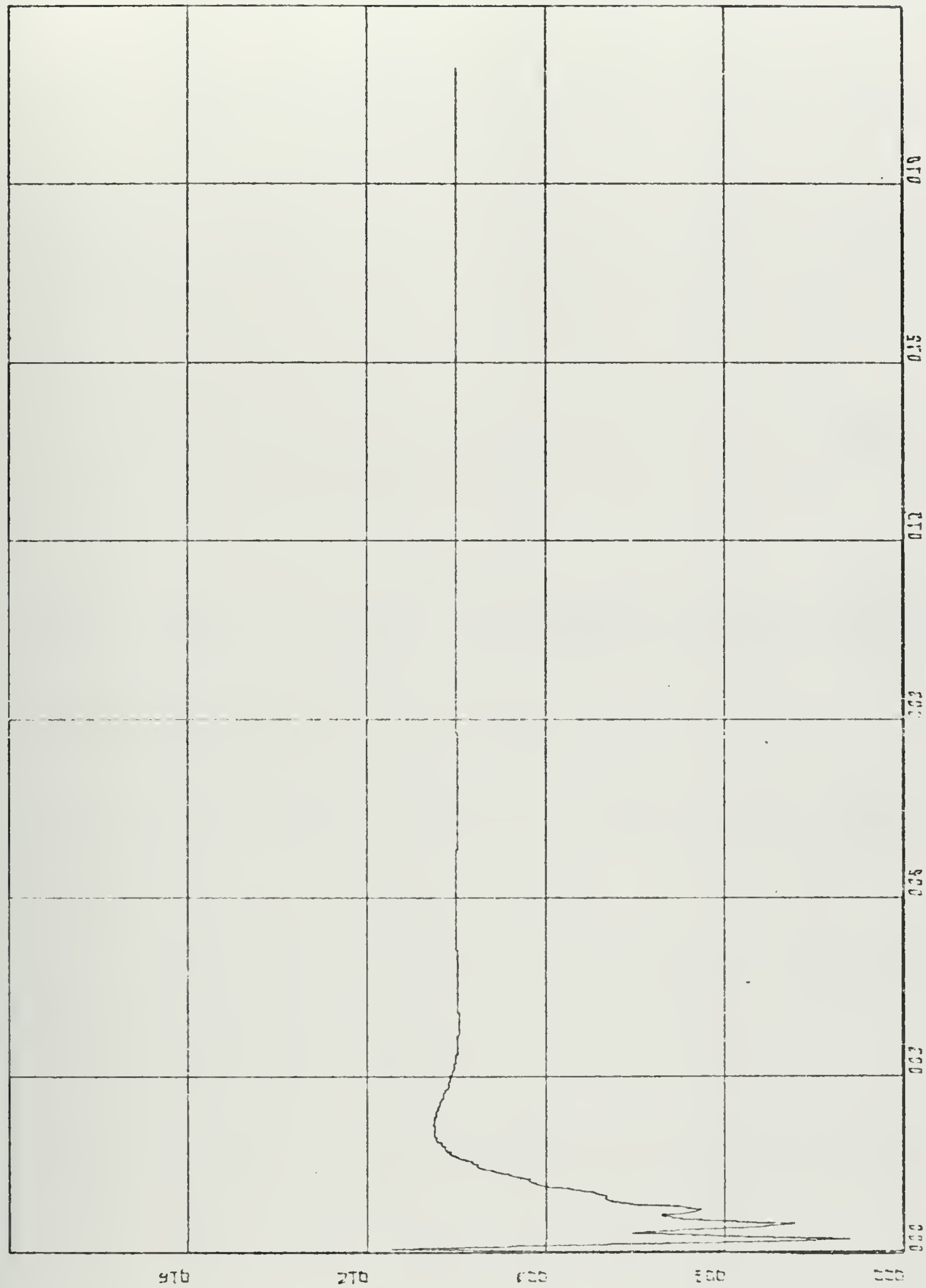


Scale: x and y = 1.0/in

Figure 3.18

Root Locus of Compensated 4th-order System

$\alpha = 1.0, \quad \beta = 0.0$

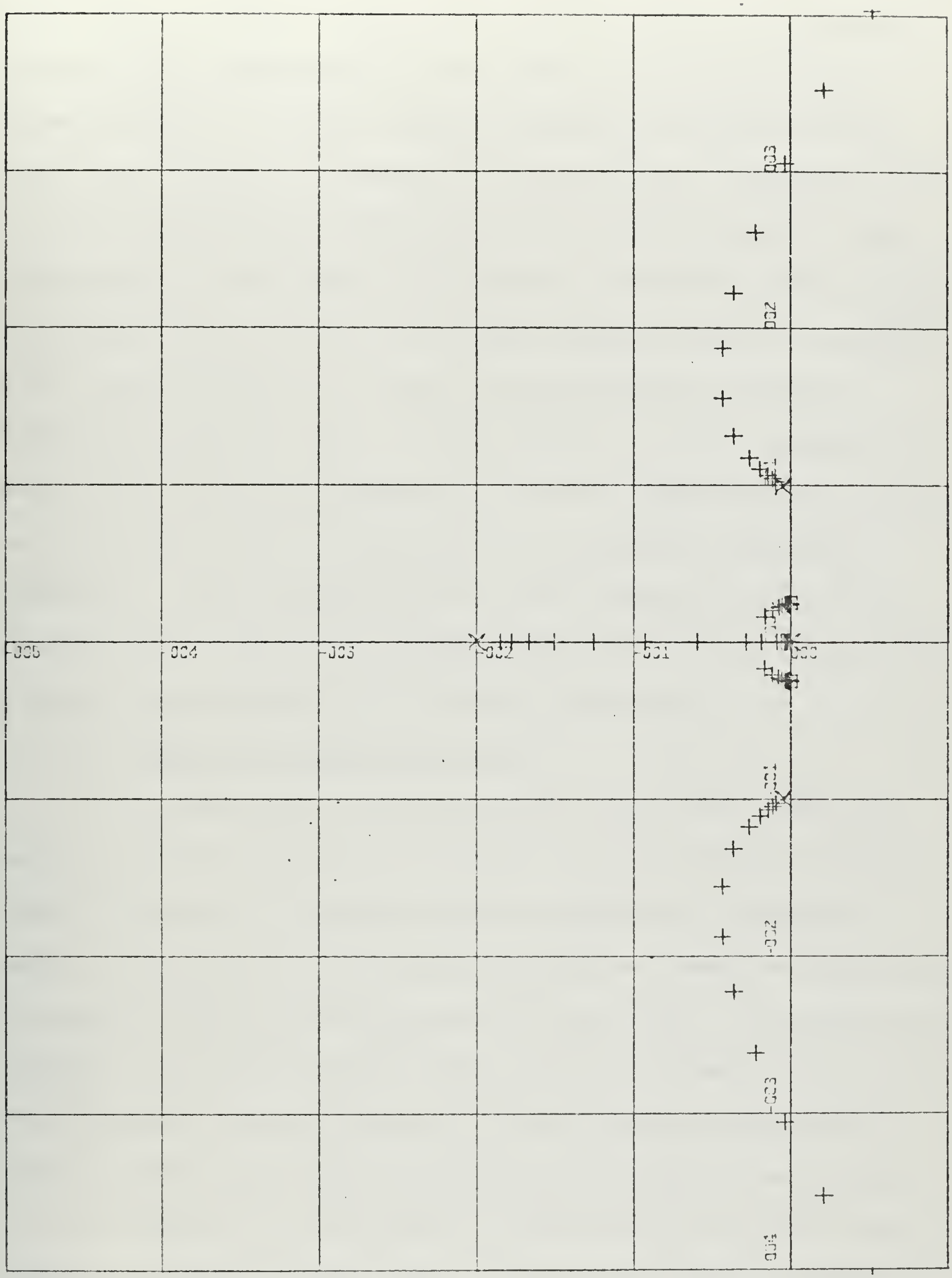


Scale: x = 30. sec/in y = 0.4 units/in

Figure 3.19

Transient Response of Compensated 4th-order System

$\alpha = 16.0, \quad \beta = 0.0$



Scale: x and y = 1.0/in

Figure 3.20

Root Locus of Compensated 4th-order System

$\alpha = 16.0, \quad \beta = 0.0$

of α and β. Such an indication can only come from the root locus of the compensated system taking into account the selected gain. If the transient response obtained for a particular choice of α and β is not suitable, the root locus of the corresponding system will give an indication of why this should be the case. Relocation of the zeros on the root locus may lead to a more suitable response and the selected zero location may be checked on the parameter plane to determine that stability of the system is still ensured. It may, therefore, be necessary to use both the parameter plane and the root locus of the system to predict a suitable response. The parameter plane alone, however, will still indicate the location of compensating zeros to give, at least, a first approximation to a desired response.

3. <u>The Use of Round-off Poles</u>

In practical engineering the use of the round-off pole is widely accepted as a means of stabilizing a system which is basically unstable due to mechanical resonance peaks. The effect of a round-off pole can best be illustrated on a Bode diagram such as shown in Fig. 3.21. If an additional pole is introduced into the system at a frequency less than the undamped natural frequency of the complex poles, the value of the magnitude of the resonant peak is decreased. If the reduction in magnitude is sufficient to ensure that the resonant peak does not cross the zero dB axis, the stability of the system will be assured.

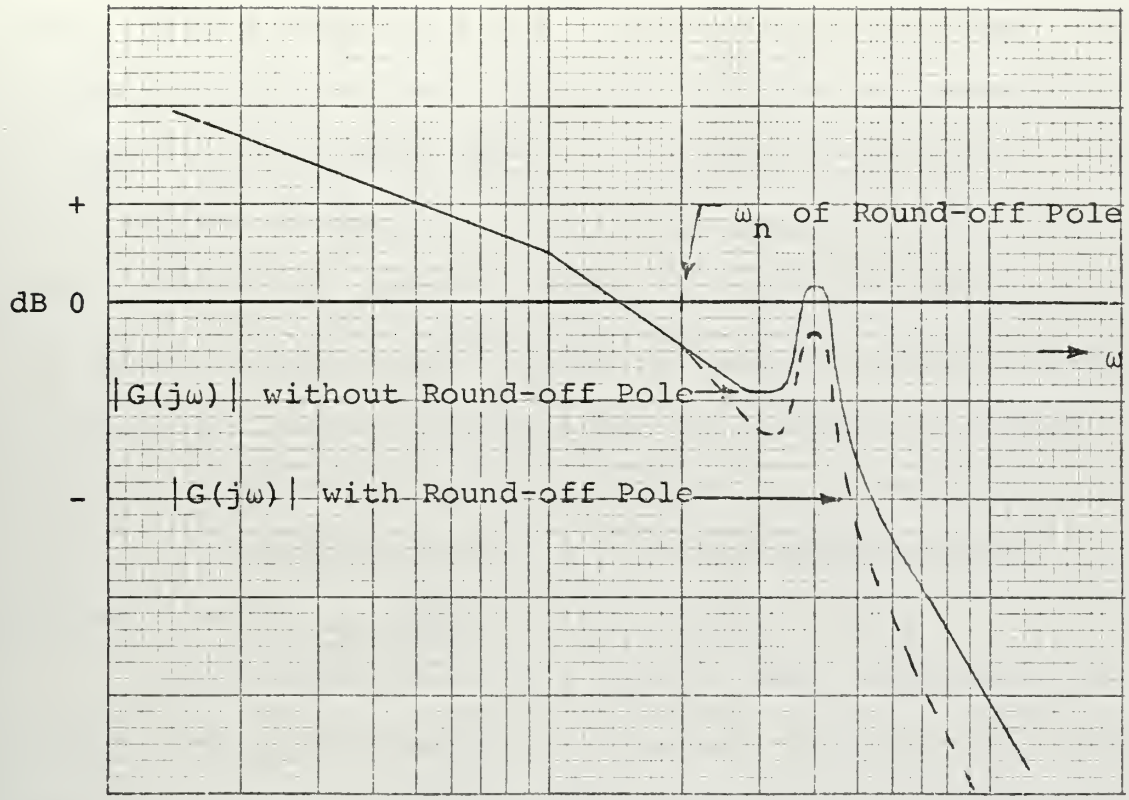


Figure 3.21

Bode Diagram Representation of the effect of Round-off Poles

To illustrate the effect of a round-off pole, an additional pole at s = -0.5 was introduced into the 4th-order system described previously. The uncompensated system transfer function becomes, therefore,

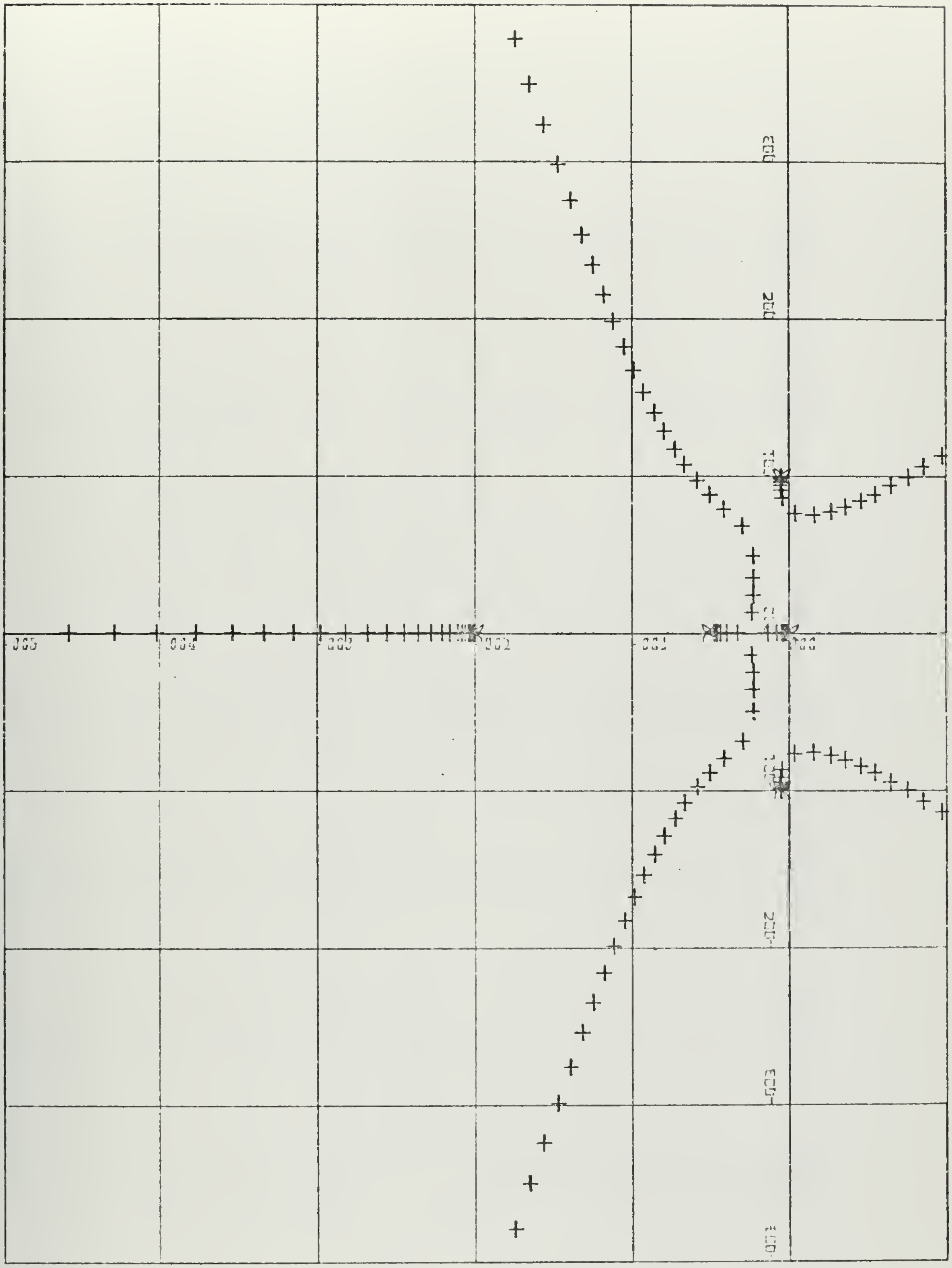
$$G(s) = \frac{K}{s(s+0.5)(s+2)(s^2+0.1\ s+1)} \quad . \qquad (3\text{-}14)$$

The value of K at the limit of stability has been increased to 0.619 with a corresponding increase in the value of the error coefficient, Kv, at the limit of stability to 0.619.

When compared with the value of the error coefficient of the 4th-order system at the limit of stability an increase of 525 percent is allowed before the system described by equation 3-14 becomes unstable. This is illustrated in Fig. 3.22, the root locus of the system. Since the root loci emerging from the complex poles curve towards the origin before crossing the imaginary axis a larger value of gain may be applied to the system before the limit of stability is reached. For a value of Kv equal to 0.177, as in the previous example, the system is stable as shown in Fig. 3.23.

The introduction of a complex zero compensator of the form described by equation 3-13 is not required to stabilize the system since it is already stable, but may be used to modify the system so that a more suitable transient response can be achieved. The parameter plane curves for this system at a constant value of Kv equal to 0.177 are shown in Fig. 3.24. The stable region is defined by the zeta = 0 curve for positive values of α. The parameter plane curves immediately show that complex zeros would have to be introduced primarily in the right half of the 's' plane in order to bring the system to the limit of stability at the given value of gain.

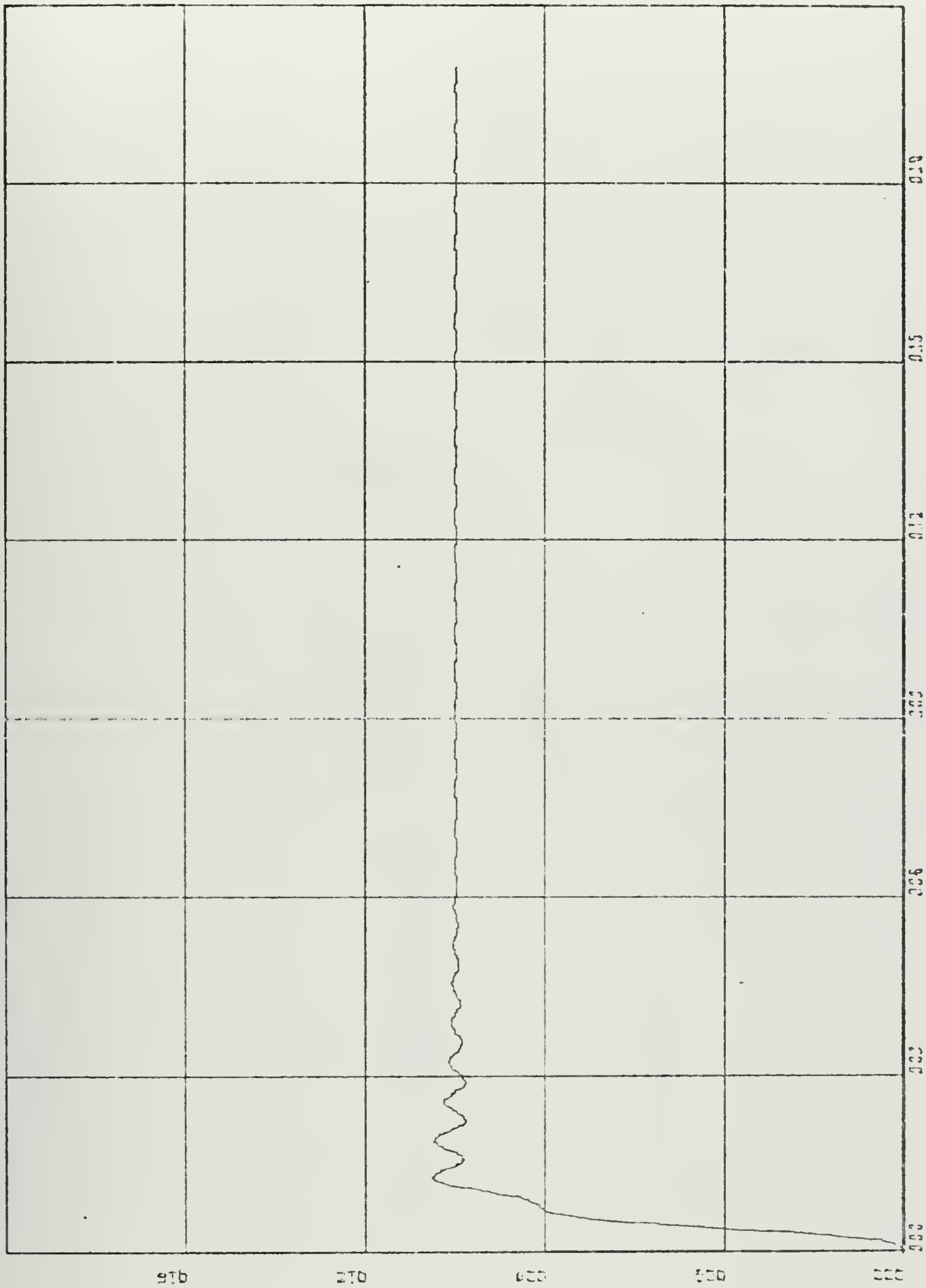
An inspection of the curves of Fig. 3.24 would lead to the conclusion that the optimum response of the system could be expected for values of α and β in the neighborhood of 1.0 and -1.5 respectively, which incidentally give rise to two real zeros. The transient response for these values of α and β is shown in Fig. 3.25 and the corresponding root locus in Fig. 3.26. A comparison of Figs. 3.23 and 3.25 indicates



Scale: x and y 1.0/in

Figure 3.22

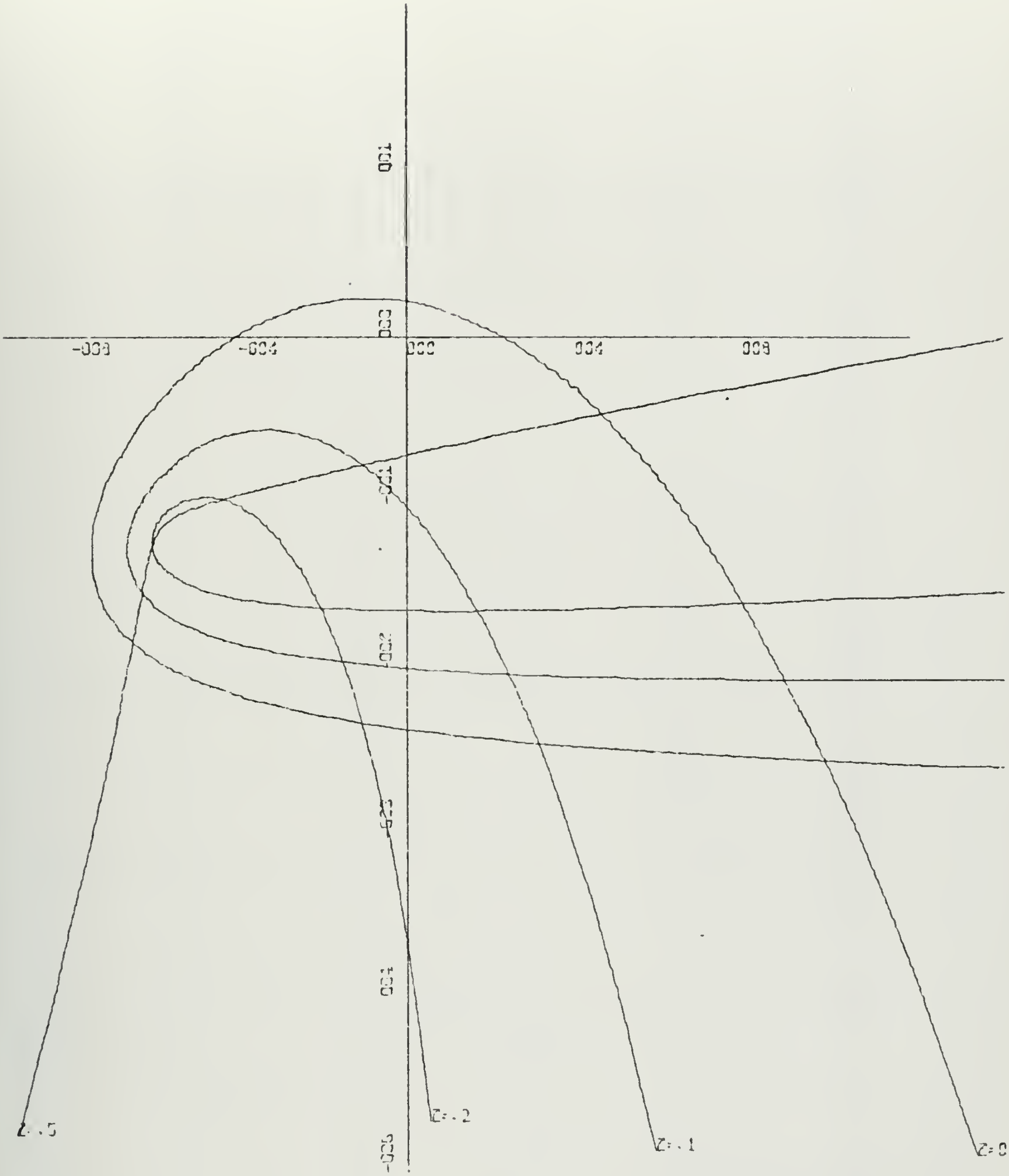
Root Locus of Uncompensated System with Round-off Pole



Scale: x = 30. sec/in, y = 0.4 units/in

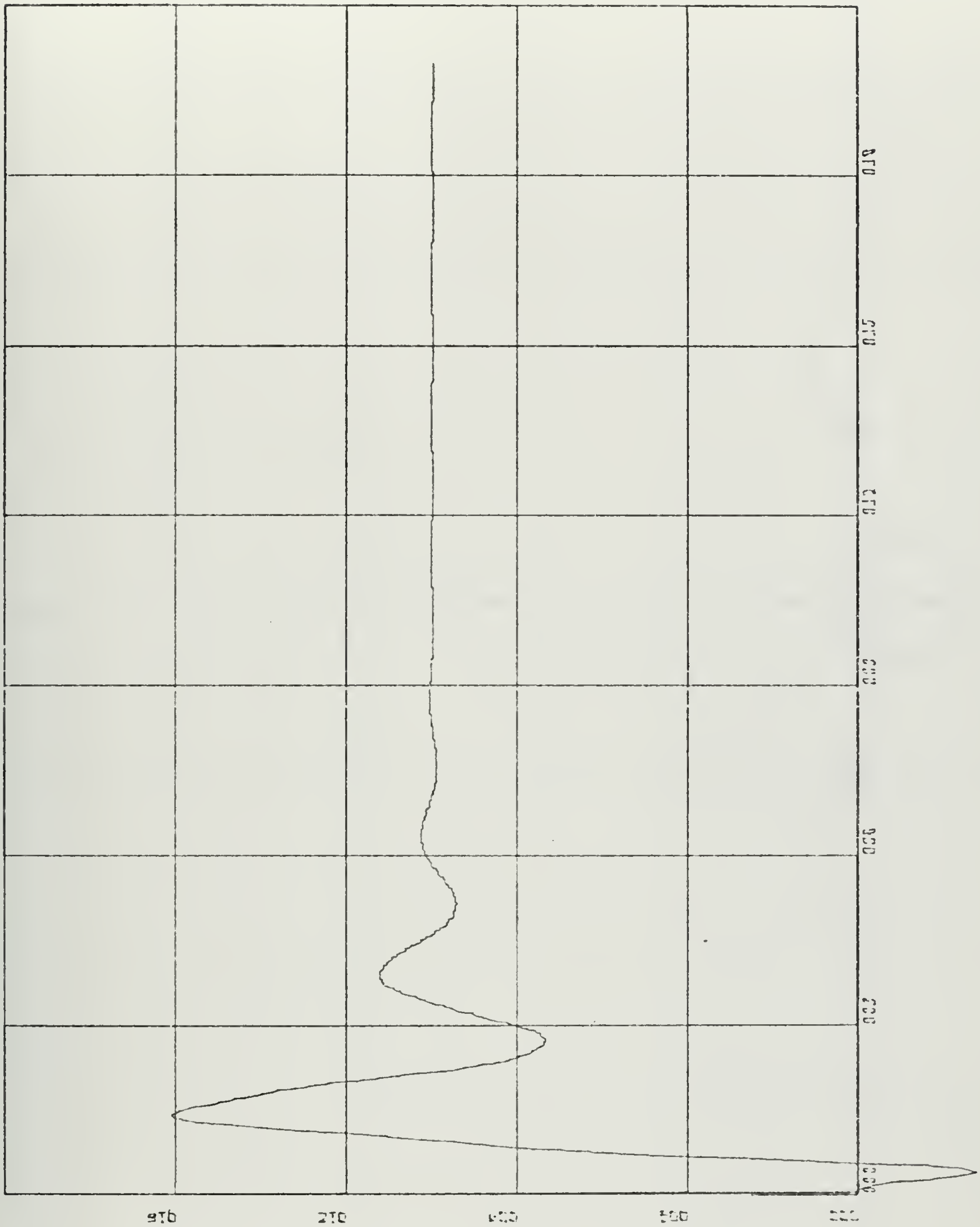
Figure 3.23

Transient Response of Uncompensated System with Round-off Pole



Scale: x = 4.0/in, y = 1.0/in

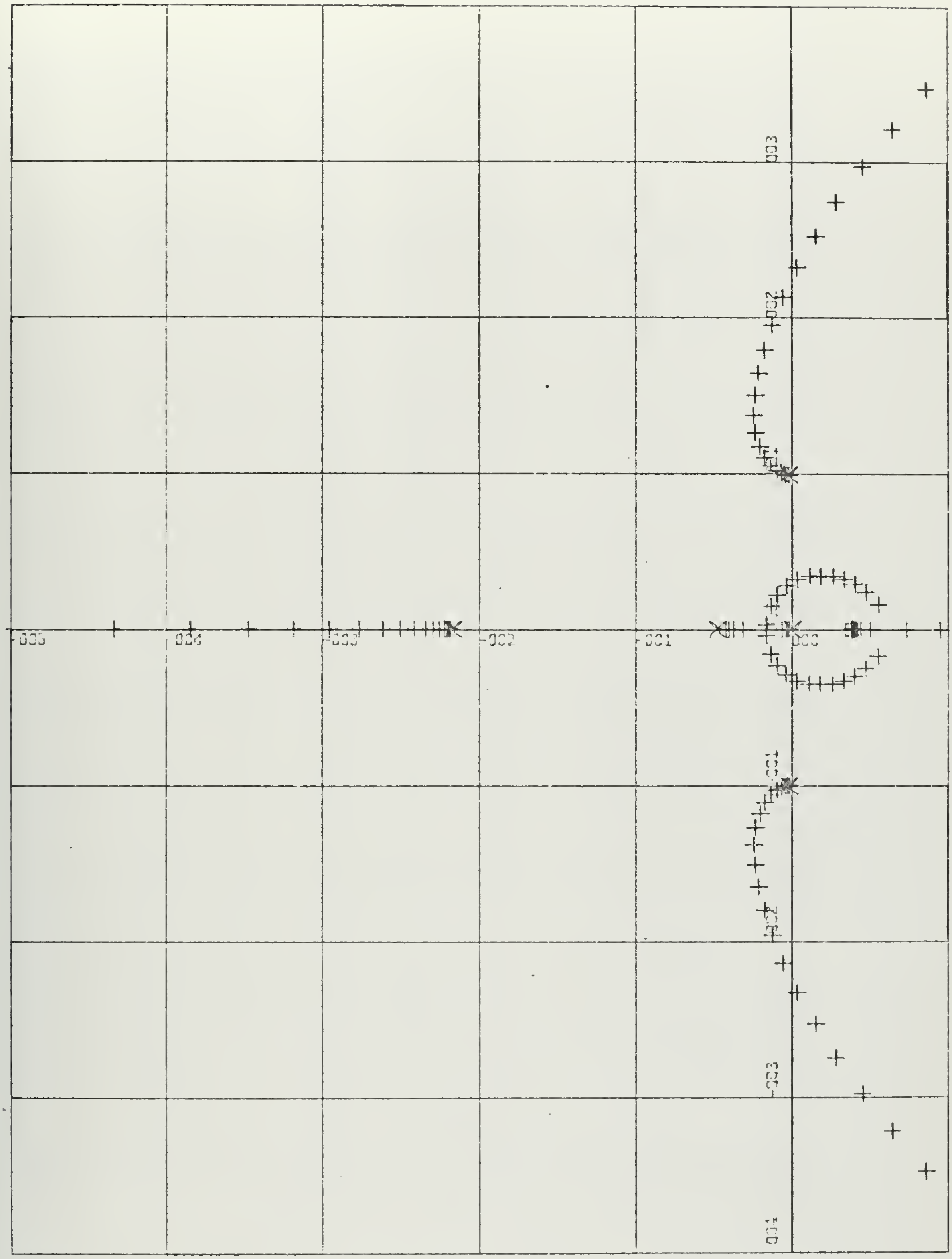
Figure 3.24
Parameter Plane Curves of System with Round-off Pole



Scale: x = 30. sec/in, y = 0.4 units/in

Figure 3.25

Transient Response of Compensated System with Round-off Pole $\alpha = 1.0$, $\beta = -1.5$



Scale: x and y 1.0/in

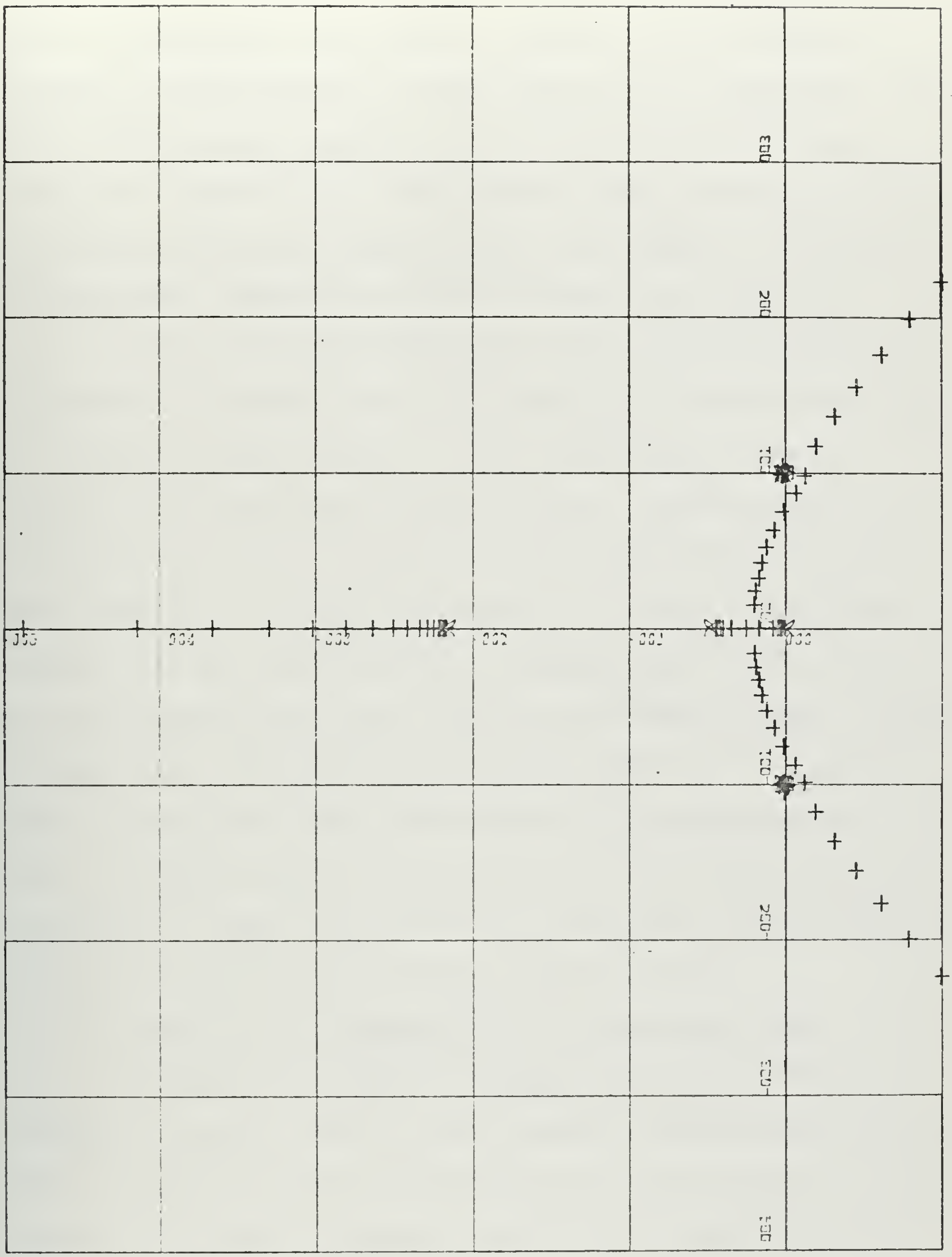
Figure 3.26

Root Locus of Compensated System with Round-off Pole

$\alpha = 1.0, \quad \beta = -1.5$

Scale: x = 30. sec/in, y = 0.4 units/in

Figure 3.27
Transient Response of Compensated System with Round-off Pole Exact Cancellation



Scale: x and y 1.0/in

Figure 3.28

Root Locus of Compensated System with Round-off Pole Exact Cancellation

that no significant advantage is gained in the transient response by introduction of the complex zero compensator, in fact, both maximum overshoot and settling time have been increased. Exact, or at least nearly exact, cancellation of the complex roots by complex zeros does, however, improve the transient response as shown in Figs. 3.27 and 3.28.

The use of round-off poles alone, may be preferrable to the use of complex zeros in stabilizing a system affected by resonance peaks because it is a much more economical method of achieving the desired results. Unfortunately the method is limited to systems that have a sufficiently large phase margin at the gain cross-over, in terms of the Bode diagram, so that the reduction in phase margin introduced by the round-off pole does not drive the basic system unstable. No particular advantage is achieved by the use of a combination of round-off poles and complex zero compensators. Exact, or nearly exact, cancellation of the complex poles may, of course, be used, but this would have been just as effective without the introduction of the round-off pole.

4. Analysis of a System with two Resonance Peaks

In practical control systems containing mechanical linkages or shafts, two or more resonant frequencies are often encountered which may be modelled by the introduction of the appropriate number of complex poles in the transfer function of the system. The feasibility of compensating a system with two resonant peaks by the use of one pair of complex zeros is investigated in the following paragraphs.

The transfer function of the system containing two pairs of complex poles is given by:

$$G(s) = \frac{K(s+2)}{s(s+.5)(s+5)(s^2+0.1\ s+1)(s^2+0.3\ s+4)} \ . \qquad (3\text{-}15)$$

The root locus of the system described by equation 3-15 is shown in Fig. 3.29. The value of K at the limit of stability was found to be 1.11 and corresponds to a value of Kv equal to 0.222. Adding a complex zero compensator of the form described by equation 3-13, and increasing the value of Kv by 50 percent to 0.333 results in the parameter plane analysis shown in Fig. 3.30. As previously explained the stable region is bounded by the zeta = 0 curve for positive values of α.

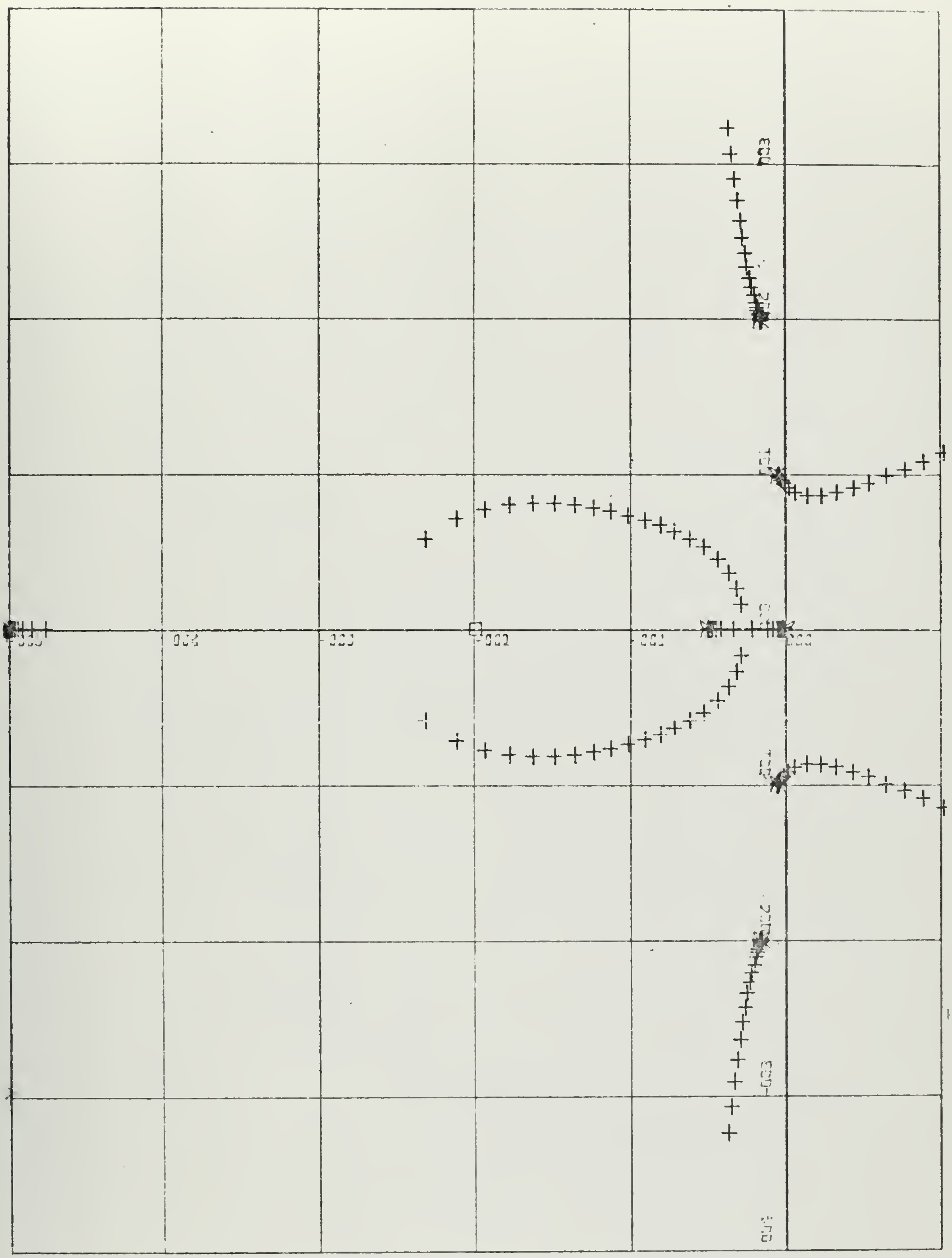
To guarantee stability of the system, values of α and β must be chosen which lie within the stability region. As a first approximation a value of α and β which falls within the region enclosed by the zeta = 0.2 curve should provide a transient response which is most acceptable. The transient response for values of α and β equal to 0.07 and -0.7 respectively is shown in Fig. 3.31 and the associated root locus in Fig. 3.32. It is obvious from Fig. 3.31 that, although the settling time may be acceptable, the maximum overshoot would appear to be excessive. For values of α and β equal to 0.2 and -0.2 respectively the transient response is shown in Fig. 3.33 and the corresponding root locus in Fig. 3.34. As expected for these values of α and β, the system is less damped so that the settling time has been increased, but the maximum overshoot has been decreased to an acceptable level.

Clearly a compromise between the two chosen values of α and β is indicated to obtain the optimum response under the given conditions.

Since stability was achieved by the use of two real zeros rather than a pair of complex ones for the choice of α and β as shown in Fig. 3.32, it may be economically advantageous in this case to chose values of α and β which will result in two real zeros. The parameter plane will, of course, indicate what range of values of α and β is permissible.

It should also be pointed out that cancellation of one pair of complex poles is no longer feasible and, in fact, prohibited since an unstable system will result from such an action. The parameter plane curves indicate this clearly since only values of α between 0.05 and 0.22 for $\beta = 0$, which correspond to ω_n between 4.47 and 2.13 for $\zeta = 0$, are within the stable region.

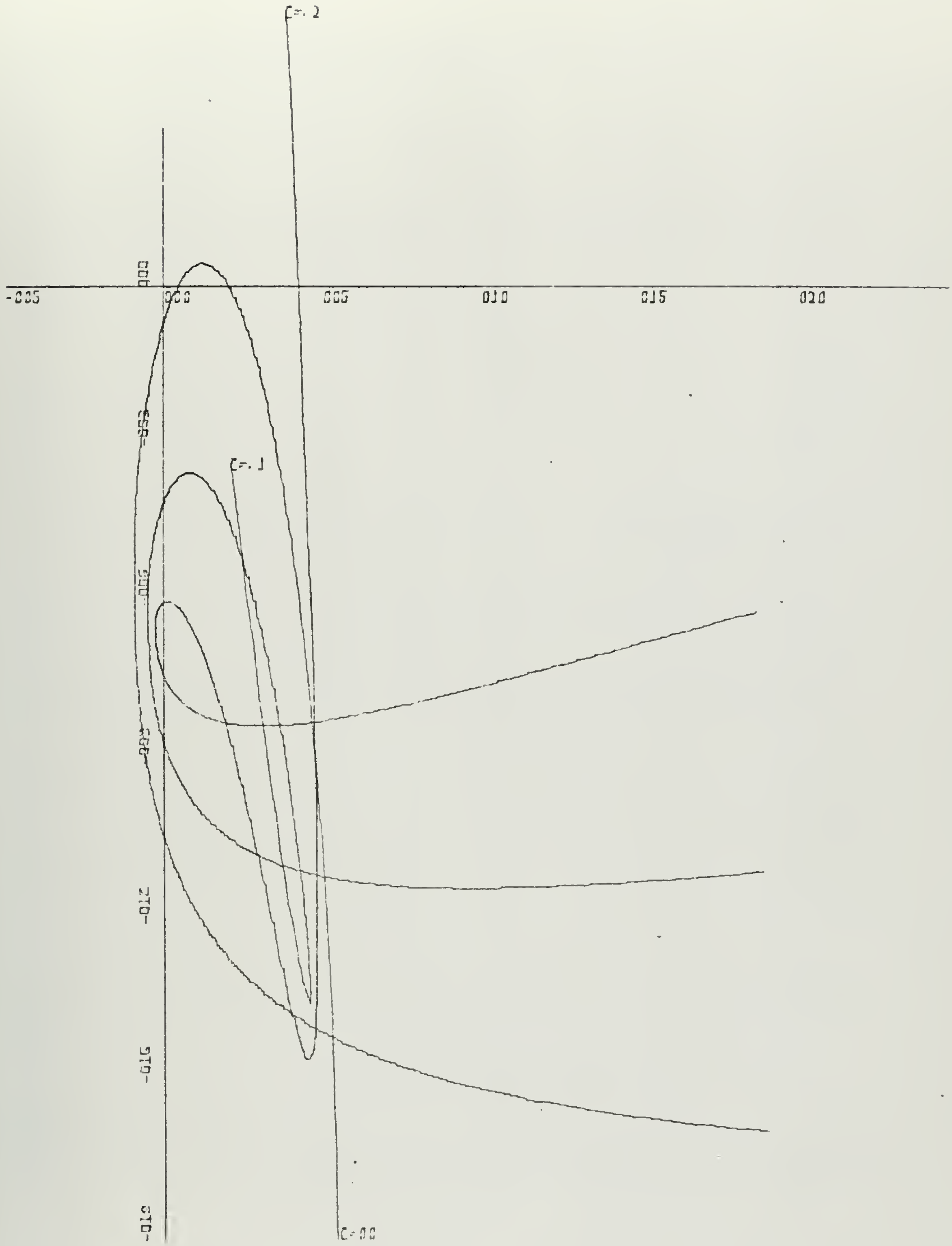
A system with only one resonance peak can, of course, always be stabilized by exact or nearly exact cancellation of the complex pole and the parameter plane analysis may not be required. For systems with two or more resonance peaks, however, cancellation of one pair of complex poles with one pair of complex zeros may not ensure stability, but a parameter plane analysis will immediately indicate if stability can be assured with only one pair of complex zeros and, if so, where they should be located.



Scale: x and y = 1.0/in

Figure 3.29

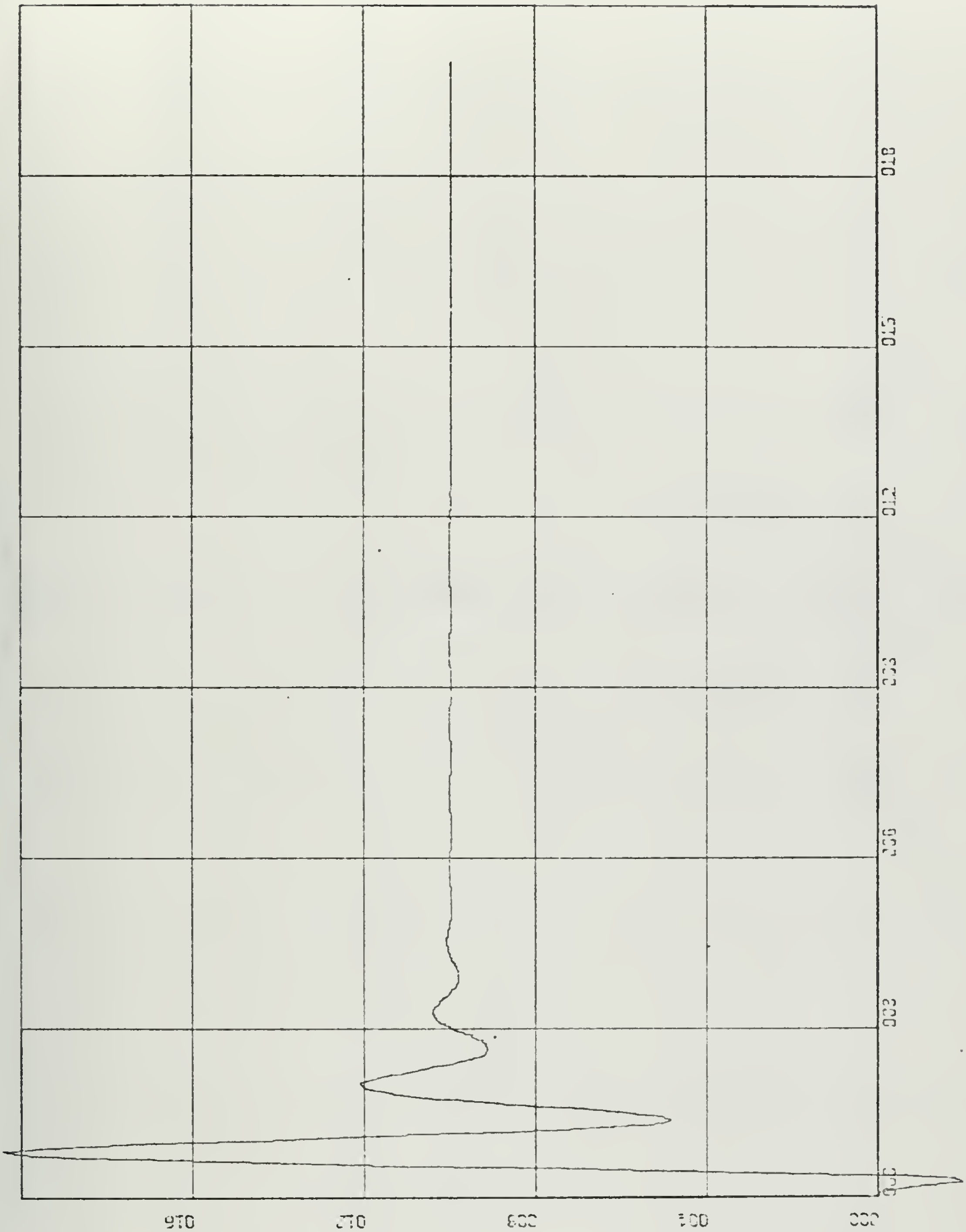
Root Locus of Uncompensated System with Two Resonance Peaks



Scale: $\alpha = 0.5/\text{in}$, $\beta = 0.3/\text{in}$

Figure 3.30

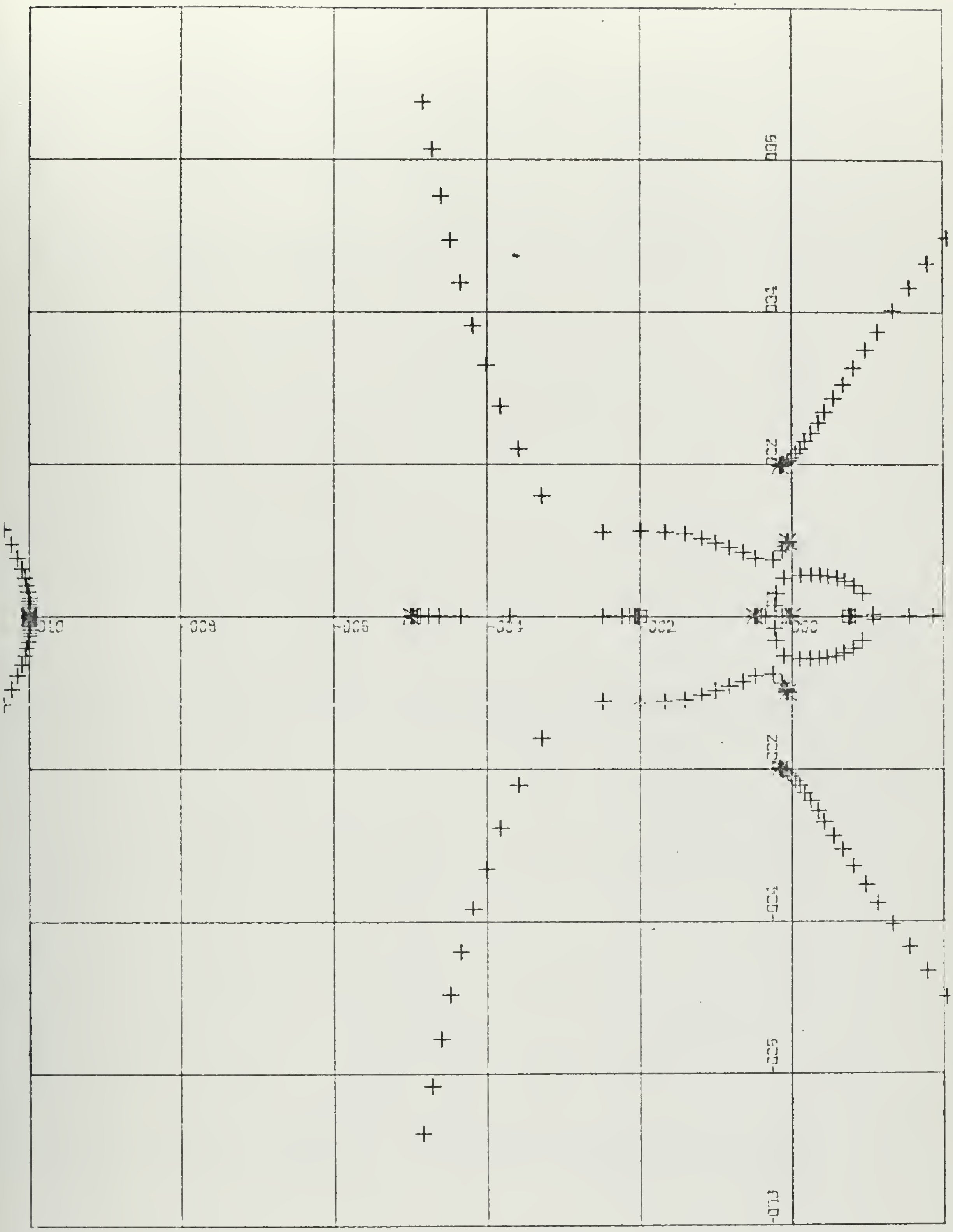
Parameter Plane Curves for System with Two Resonance Peaks



Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.31

Transient Response of Compensated System with Two Resonance Peaks $\alpha = 0.07$, $\beta = -0.7$

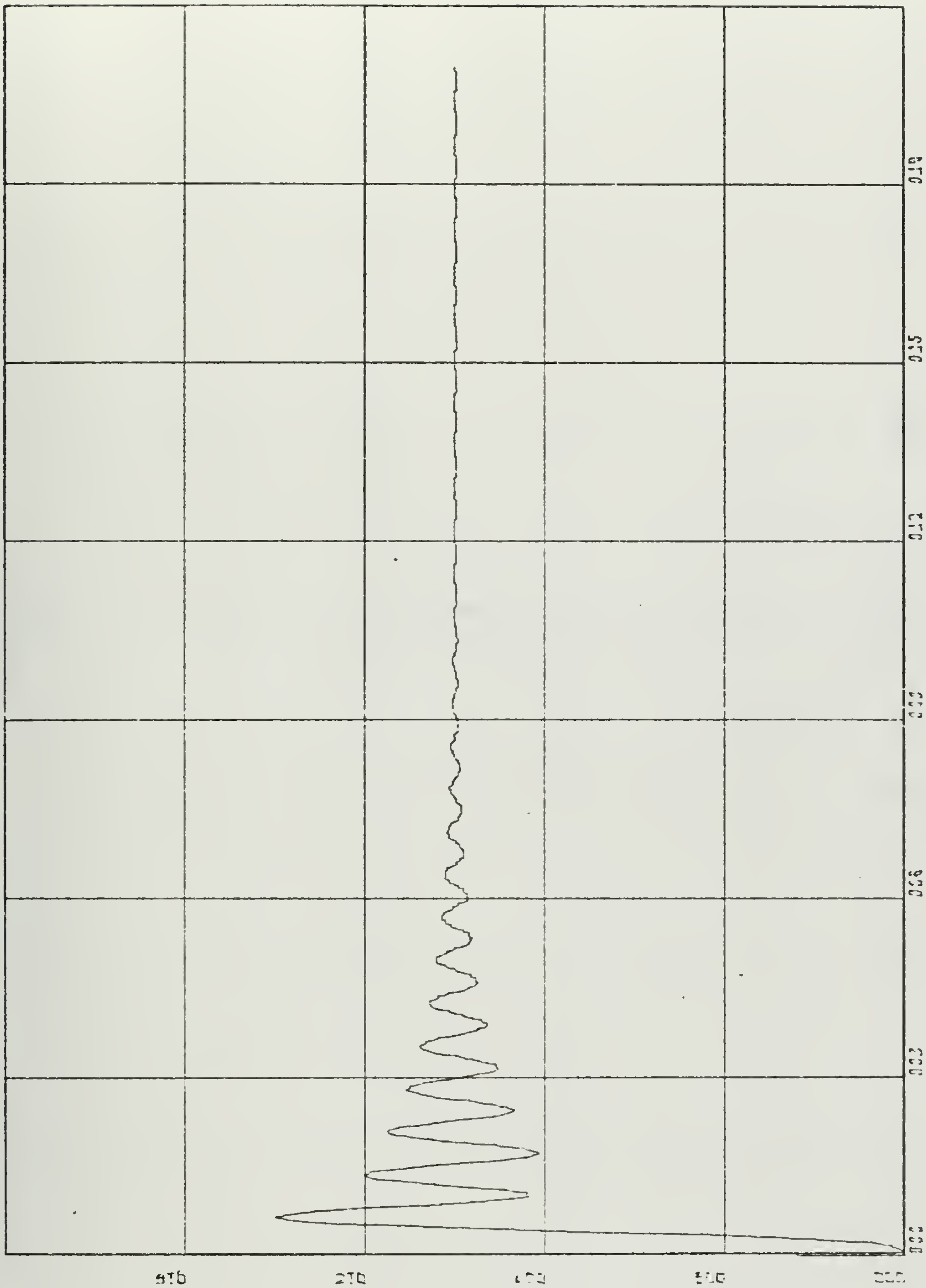


Scale: x and y = 2.0/in

Figure 3.32

Root Locus of Compensated System with Two Resonance Peaks

$\alpha = 0.07, \quad \beta = -0.7$



Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.33

Transient Response of Compensated Systed System with Two Resonance Peaks $\alpha = 0.2$, $\beta = -0.2$

Scale: x and y = 2.0/in

Figure 3.34

Root Locus of Compensated System with Two Resonance Peaks

$\alpha = 0.2, \quad \beta = -0.2$

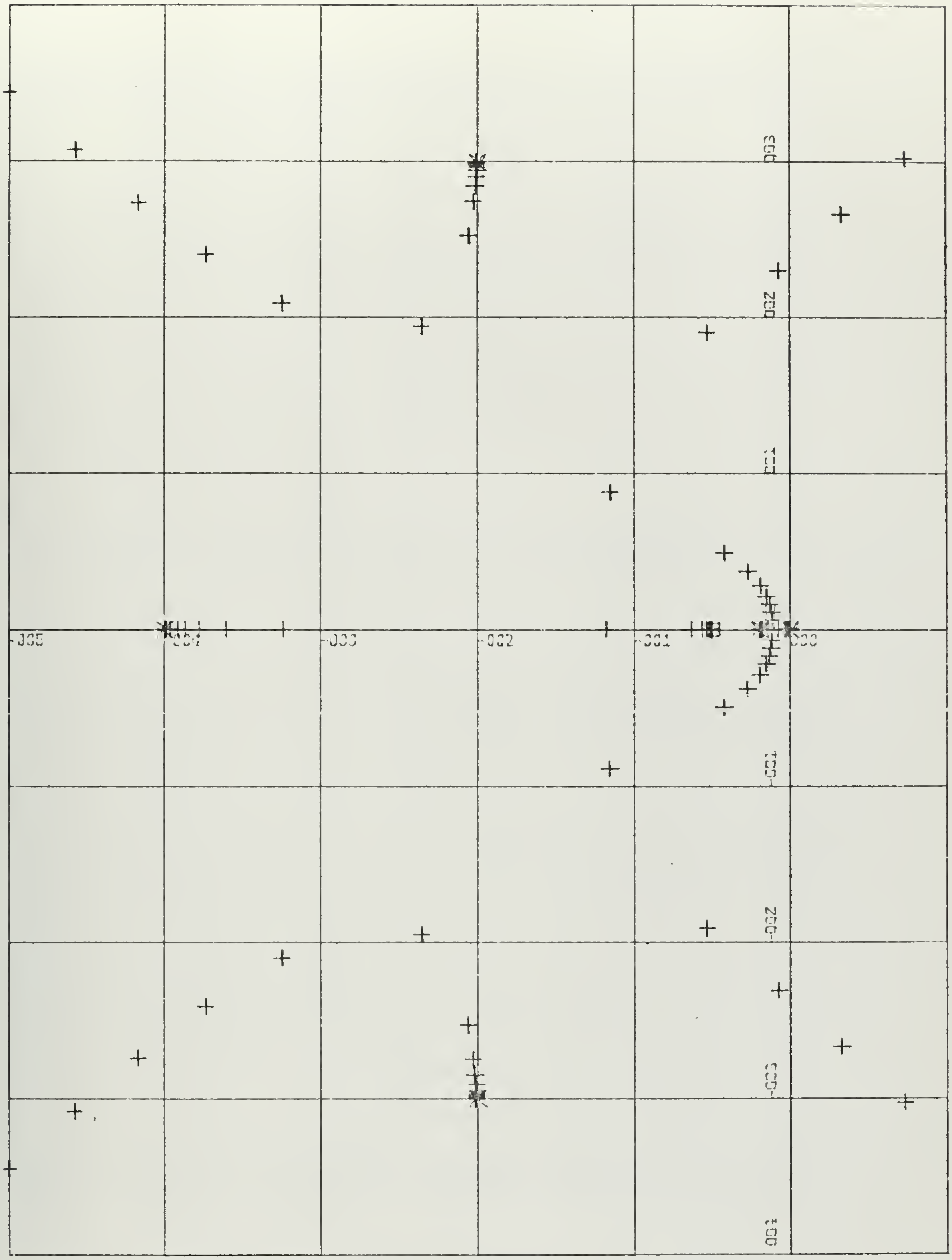
5. Analysis of Systems with Resonance-Antiresonance Doublets

Although the effect of structural resonances in practical control systems may be modelled by the introduction of a number of complex poles in the transfer function of the system as described in the previous section, a more accurate description of the system is often found by modelling the structural resonances as resonant-antiresonant doublets. The term 'doublet' is used to indicate that the characteristic of the structural resonance is such that a resonance peak combined with an anti-resonance trough appear at closely related frequencies in the frequency response of the overall system. Such a doublet can be modelled by the inclusion of a combination of one pair of complex poles and one pair of complex zeros in the system transfer function.

To illustrate the effect of structural resonances which can be modelled in the form of resonance-antiresonance doublets, a basic type 1 - 5th order system whose transfer function is,

$$G(s) = \frac{K(s+0.5)}{s(s+0.2)(s+4)(s^2+4s+13)} \qquad (3\text{-}16)$$

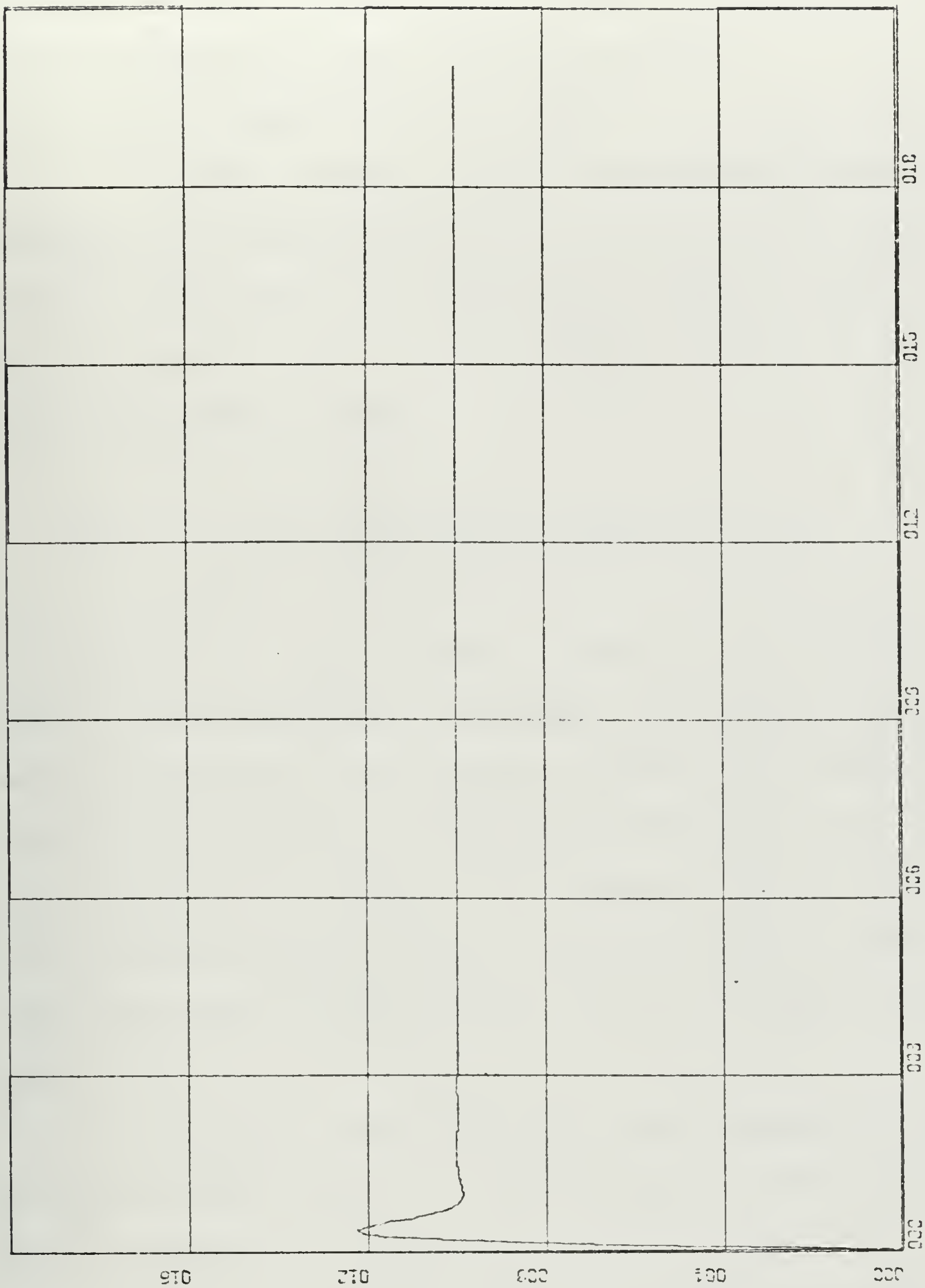
and whose corresponding root locus is shown in Fig. 3.35 will be subjected to a number of structural resonances in the form of doublets. For a selected gain, K, equal to 31.2, the error coefficient, Kv, is 1.5 and the system transient response, as shown in Fig. 3.36, was deemed acceptable from a



Scale: x and y = 1.0/in

Figure 3.35

Root Locus of 5th-order System without Structural Resonances



Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.36

Transient Response of 5th-order System without Structural Resonances Kv = 1.5

standpoint of maximum overshoot and settling time. An analysis of this system when subjected to structural resonances will be presented in the following sections.

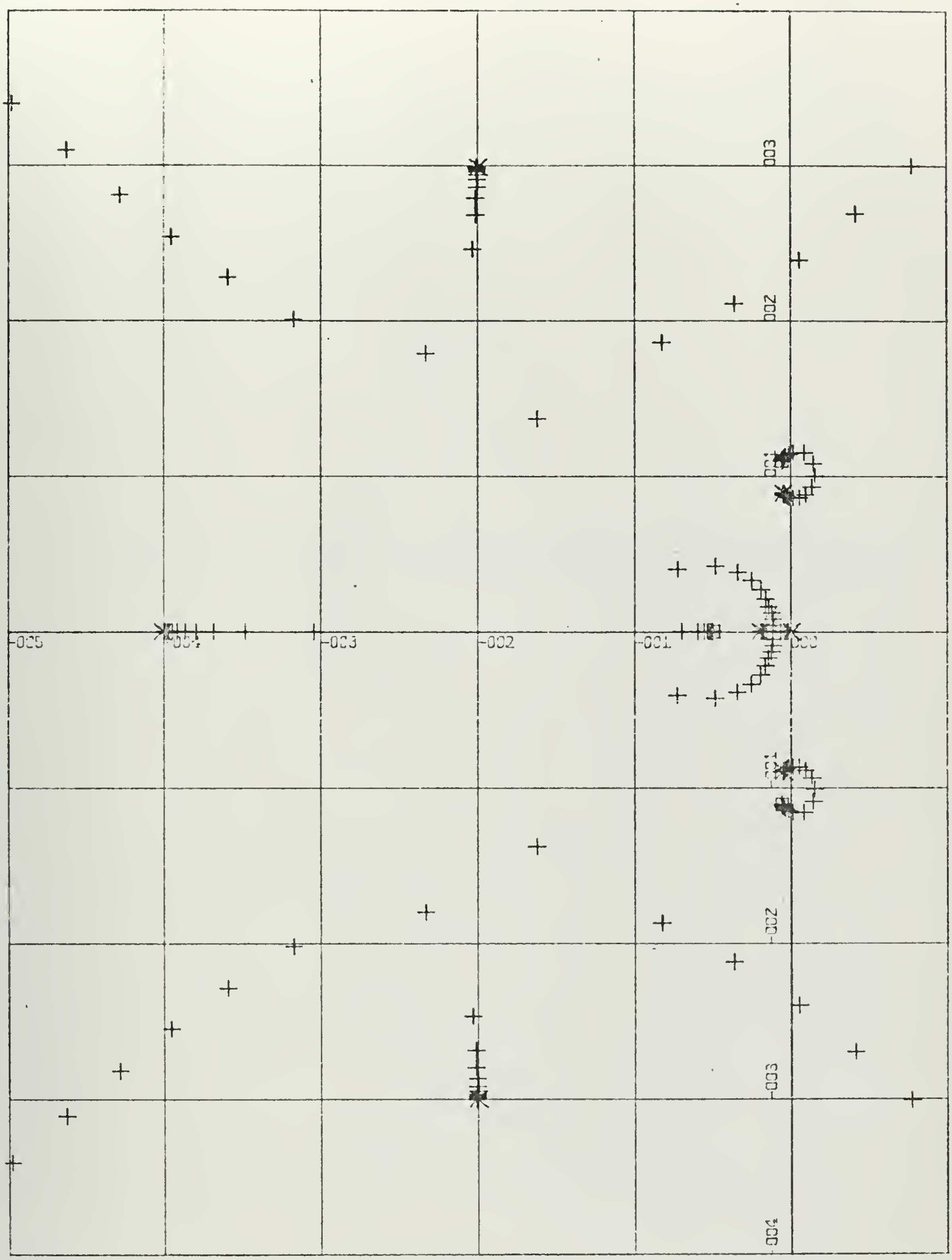
a. A System with One Doublet

The inclusion of one resonance-antiresonance doublet in the system described in the previous paragraph will result in the addition of one pair of complex zeros and one pair of complex poles to the transfer function of the system which, therefore, becomes:

$$G(s) = \frac{K(s+0.5)(s^2+0.12s+1.22)}{s(s+0.2)(s+4)(s^2+4s+13)(s^2+0.1\,s+0.8)} \quad . \qquad (3\text{-}17)$$

The root locus of this system is shown in Fig. 3.37, indicating that for the specified value of Kv the system is unstable. In order to stabilize this system a complex zero compensator of the form described by equation 3-13 was used and a parameter plane analysis for the specified value of Kv was carried out. The resultant parameter plane curves are shown in Fig. 3.38 indicating that a stable region exists, as shown by the shaded area.

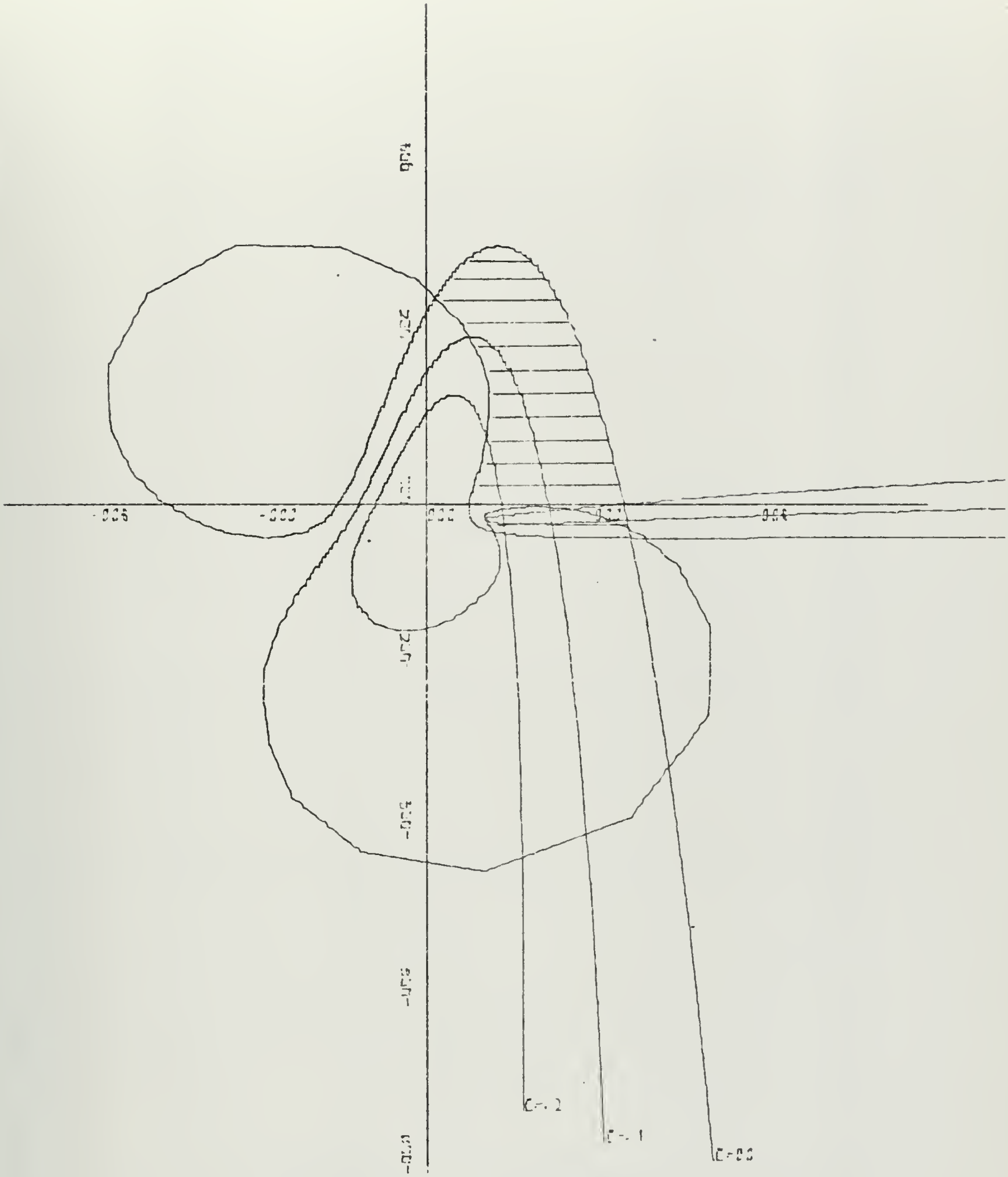
Since a portion of the area bounded by the zeta = 0.2 curve lies within the stable region, a selection of α and β from that area of intersection should result in a compensated system which has the most acceptable transient response achievable under the specified conditions. For values of α and β equal to 1.0 and 0.0 respectively, which are contained within the area of intersection, the transient response of the



Scale: x and y 1.0/in

Figure 3.37

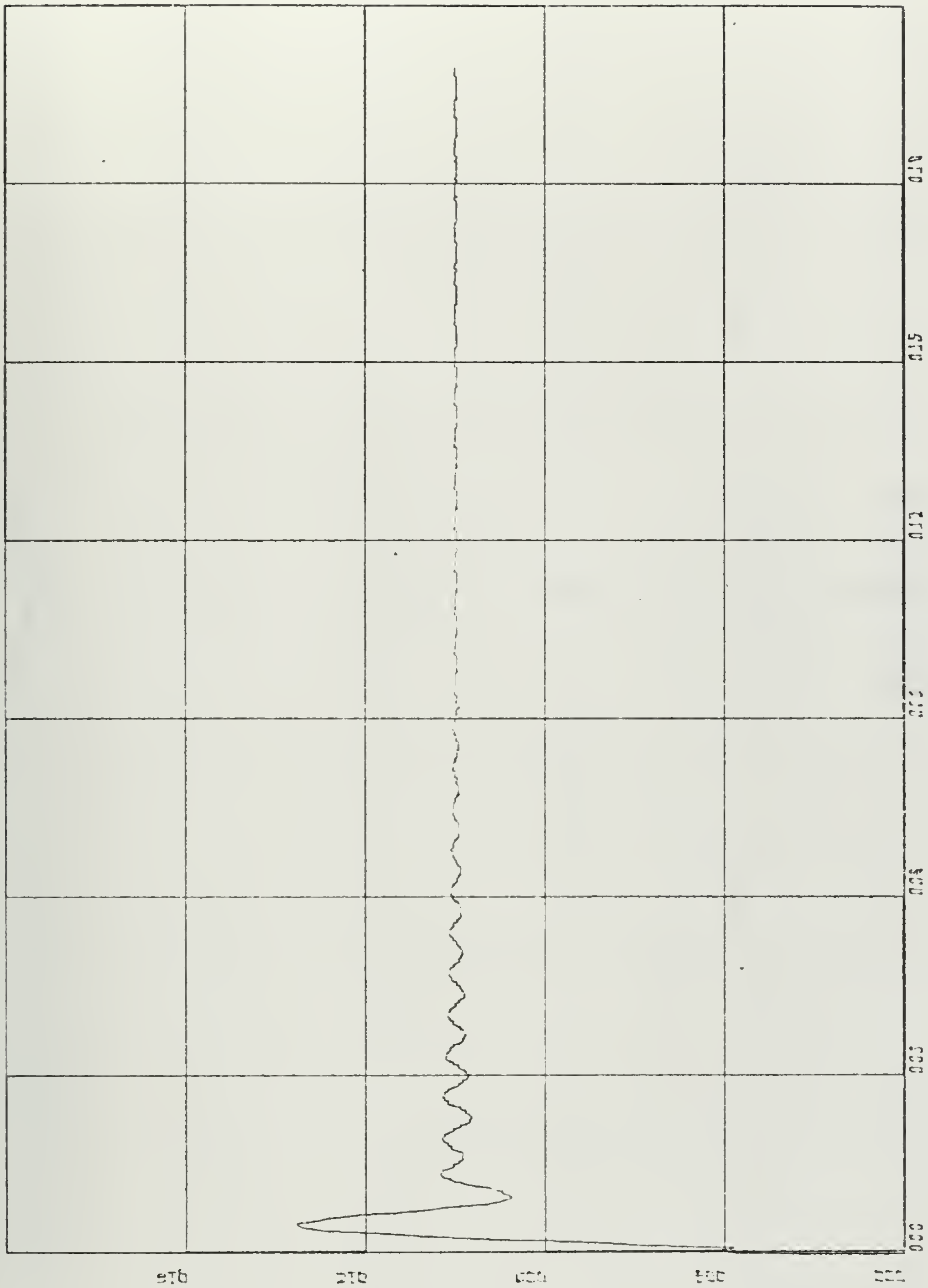
Root Locus of Uncompensated System with one Doublet



Scale: α = 3.0/in, β = 2.0/in

Figure 3.38

Parameter Plane Curves for System with one Doublet

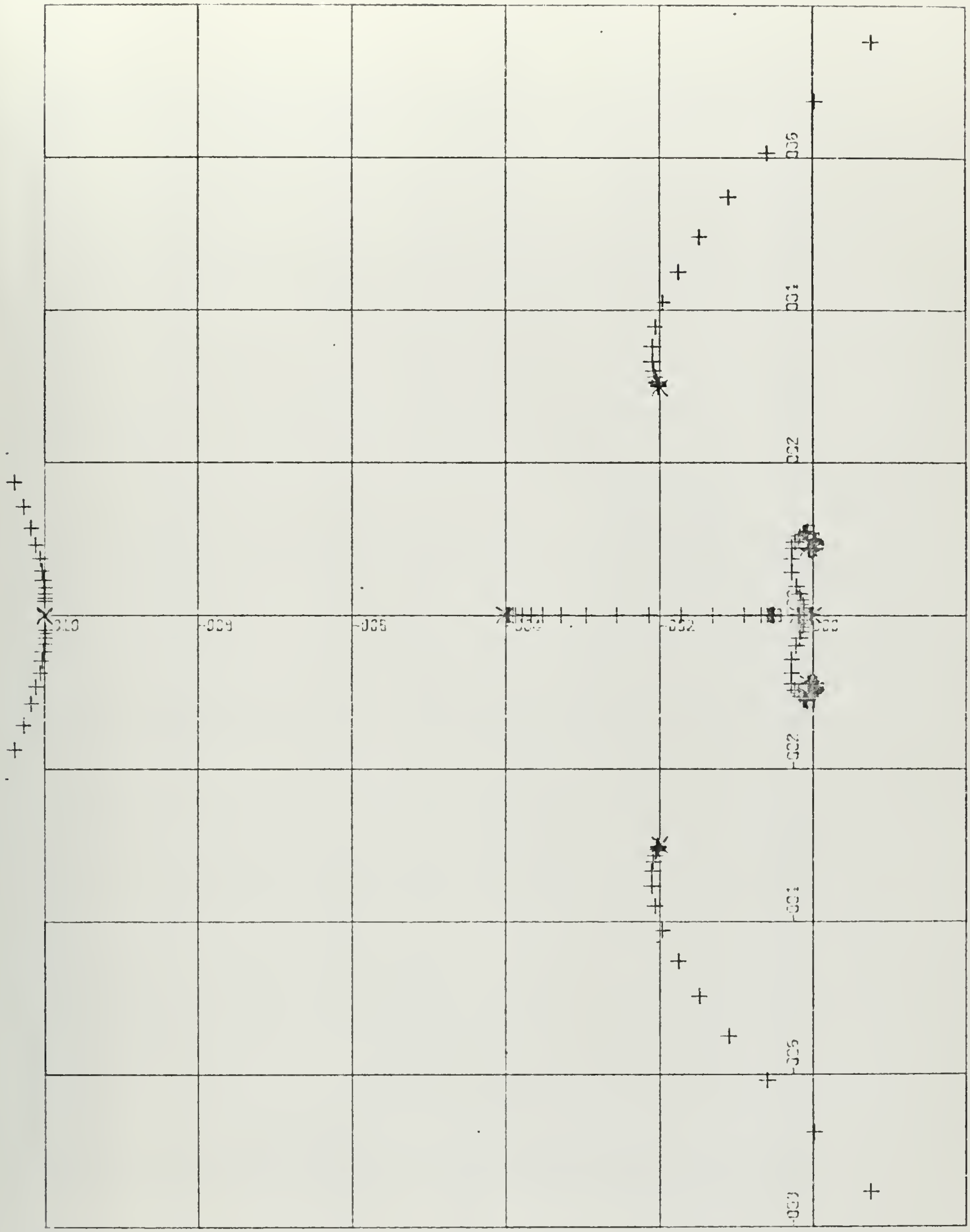


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.39

Transcient Response of Compensated System with one Doublet

$\alpha = 1.0, \quad \beta = 0.0$



Scale: x and y = 2.0/in

Figure 3.40

Root Locus of Compensated Systems with one Doublet

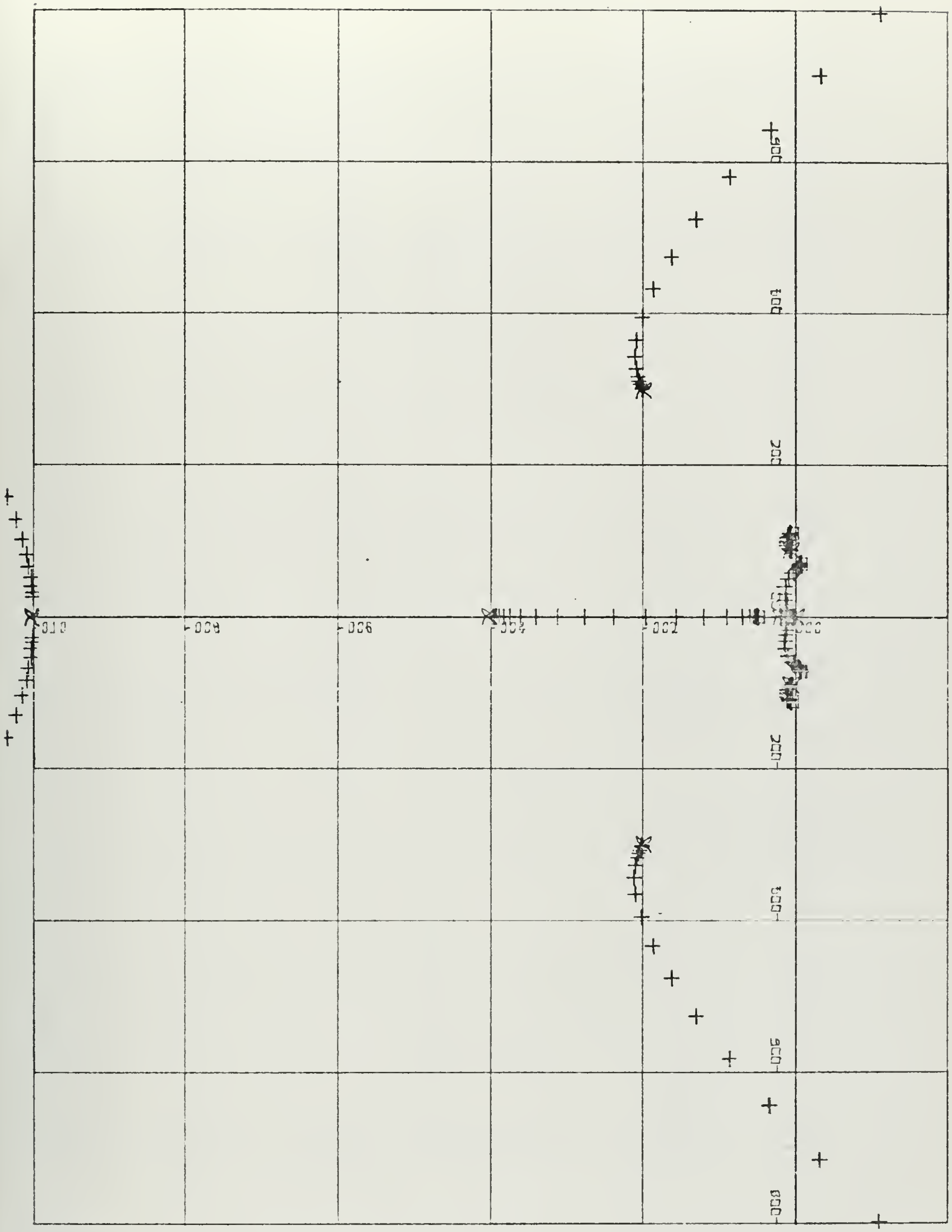
$\alpha = 1.0, \quad \beta = 0.0$

Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.41

Transient Response of Compensated System with one Doublet

$\alpha = 2.0, \quad \beta = -0.15$

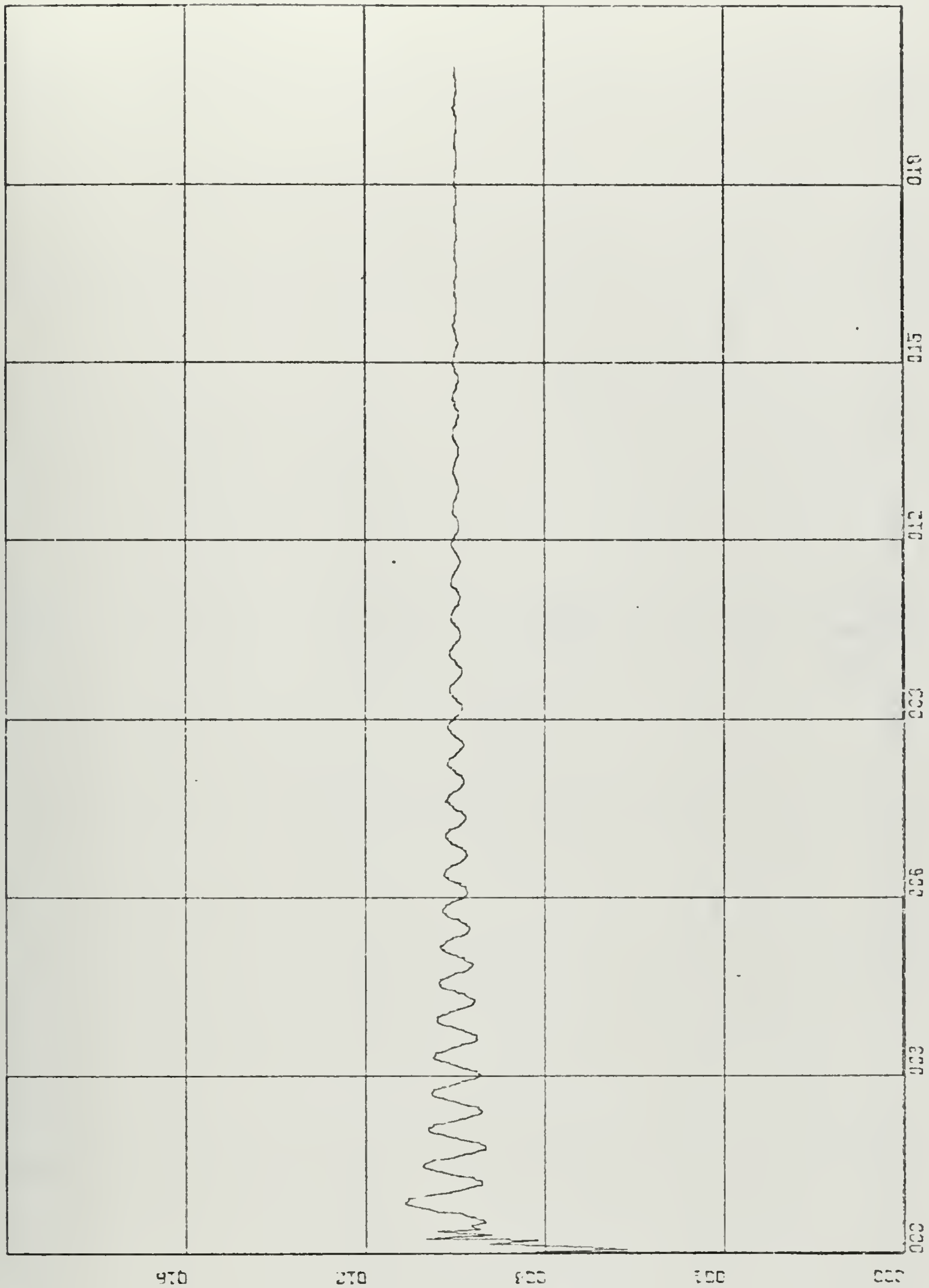


Scale: x and y = 2.0/in

Figure 3.42

Root Locus of Compensated System with one Doublet

$\alpha = 2.0, \quad \beta = -0.15$

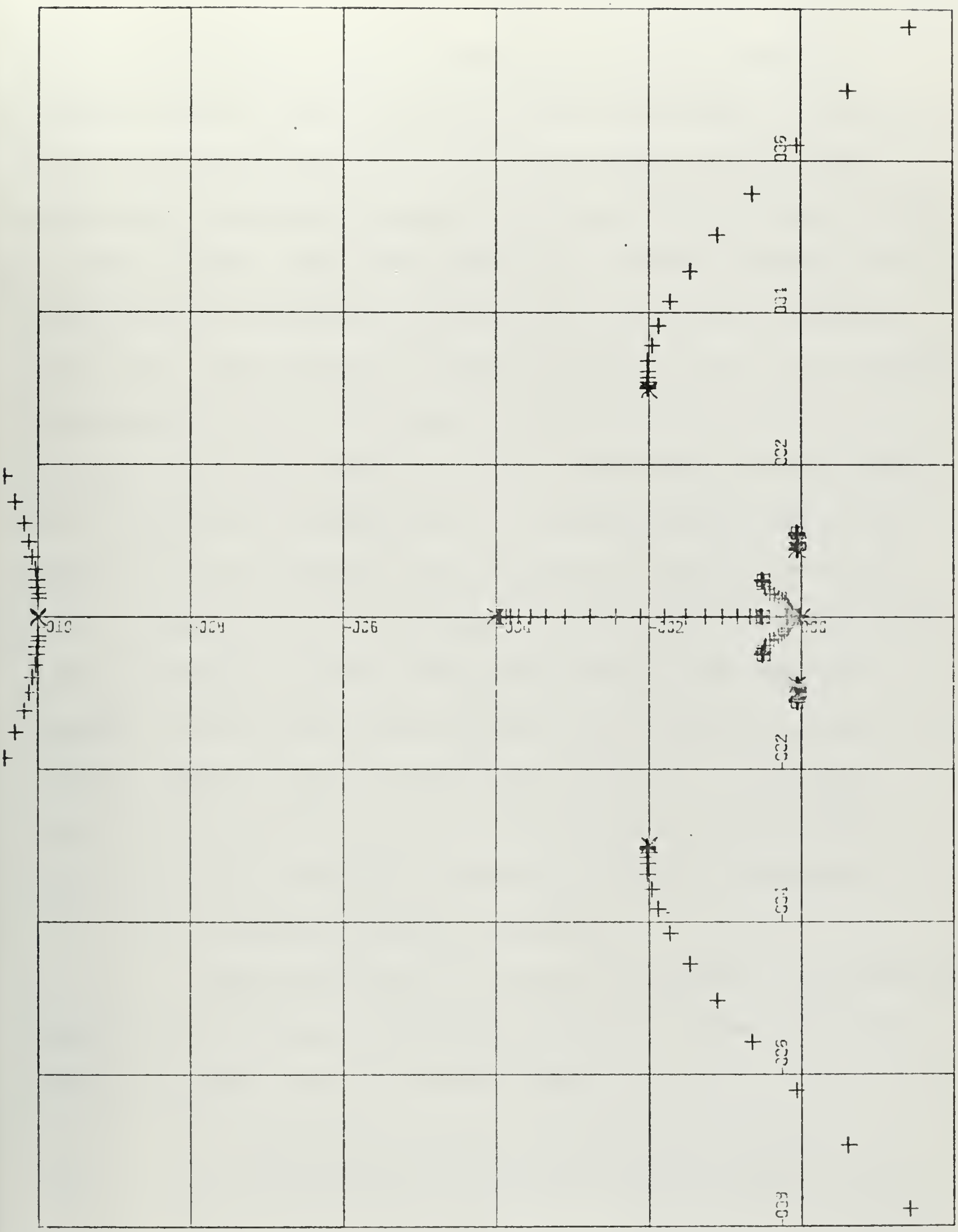


Scale: 'x = 30 sec/in, y = 0.4 units/in

Figure 3.43

Transient Response of Compensated System with one Doublet

$\alpha = 2.0$, $\beta = 1.0$



Scale: x and y = 2.0/in

Figure 3.44

Root Locus of Compensated System with one Doublet

$\alpha = 2.0, \quad \beta = 1.0$

compensated system is shown in Fig. 3.39 and the corresponding root locus in Fig. 3.40. A comparison of the transient response shown in Fig. 3.39 with those displayed in Figs. 3.41 and 3.43 would support the assumption that the most acceptable transient response is obtained by choosing values of α and β which lie within the area of intersection of the stable region and the area bounded by the largest constant zeta curve. The transient response shown in Fig. 3.39 appears acceptable as far as maximum overshoot is concerned, but displays a certain amount of small amplitude 'ringing' which may or may not be acceptable in a particular system. The ringing is not present in the transient response shown in Fig. 3.41 and, thus, a compromise between the two values of α and β selected may be indicated to obtain the optimum response of this particular system. The parameter plane curves, however, do indicate what values of α and β will stabilize the system and, to a first approximation, what values will give the most acceptable transient response.

b. A System with Two Doublets

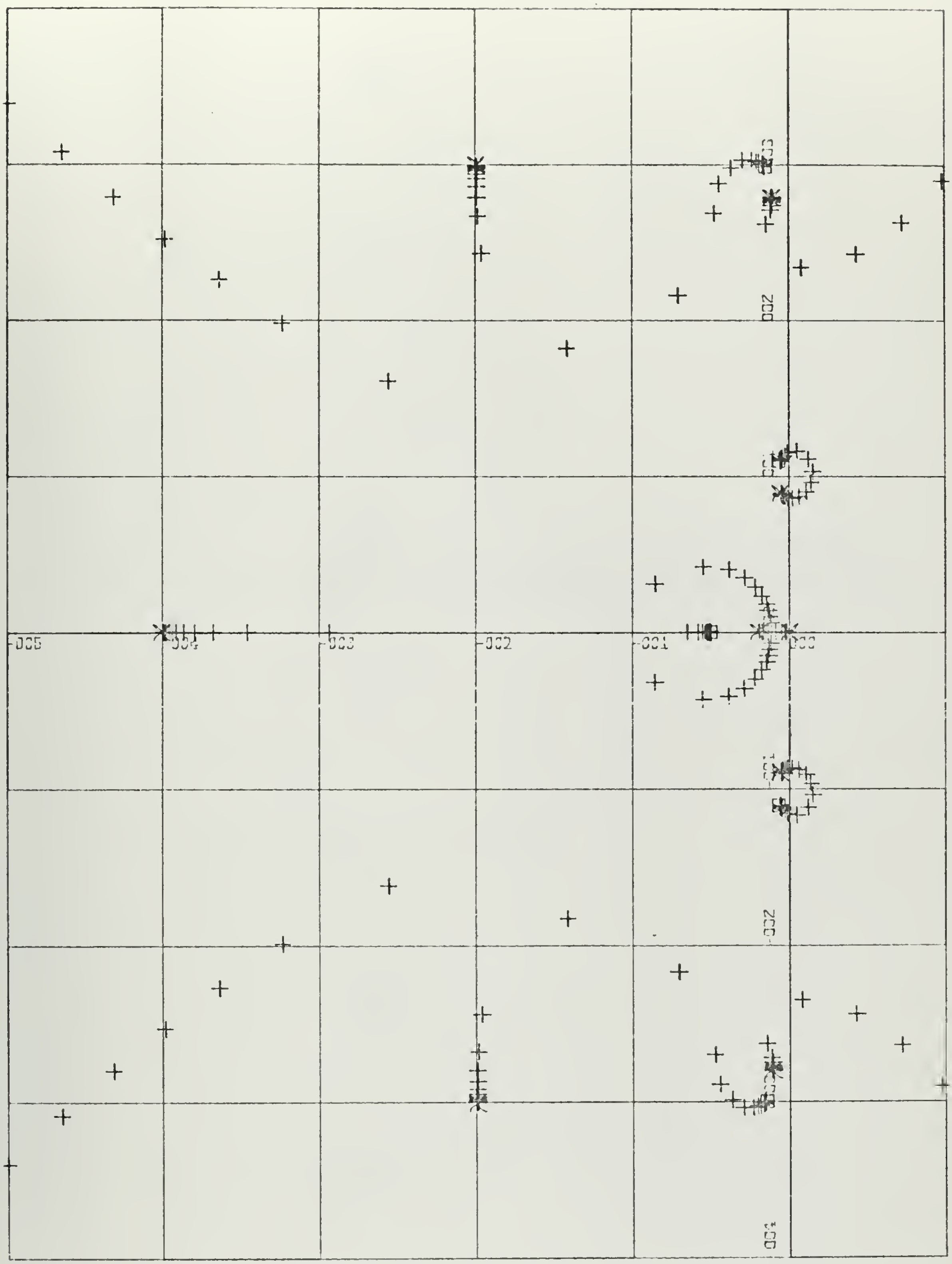
The inclusion of a second resonance - antiresonance doublet in the system described previously gives rise to a system transfer function of the form:

$$G(s) = \frac{K(s+0.5)(s^2+0.12s+1.22)(s^2+0.3s+9.0)}{s(s+0.2)(s+4)(s^2+4s+13)(s^2+0.1\,s+0.8)(s^2+0.2s+7.85)} . \tag{3.18}$$

The root locus of the system described by equation 3-18 is shown in Fig. 3.45 and again the system is unstable for a value of Kv equal to 1.5. As before, it was attempted to

stabilize the system using a complex zero compensator of the form described by equation 3-13. The resultant parameter plane analysis, shown in Fig. 3.46, again indicates a stable region closely resembling the one obtained for one doublet. Since the instability in this particular system is caused by the first doublet, as shown in Fig. 3.45, it is reasonable to assume that the system can be stabilized in a manner similar to the one used for one doublet. The similarity of the two parameter plane curves, Figs. 3.38 and 3.46, substantiates this assumption. The small differences that can be detected between the two parameter plane curves are due to the presence of the extra doublet.

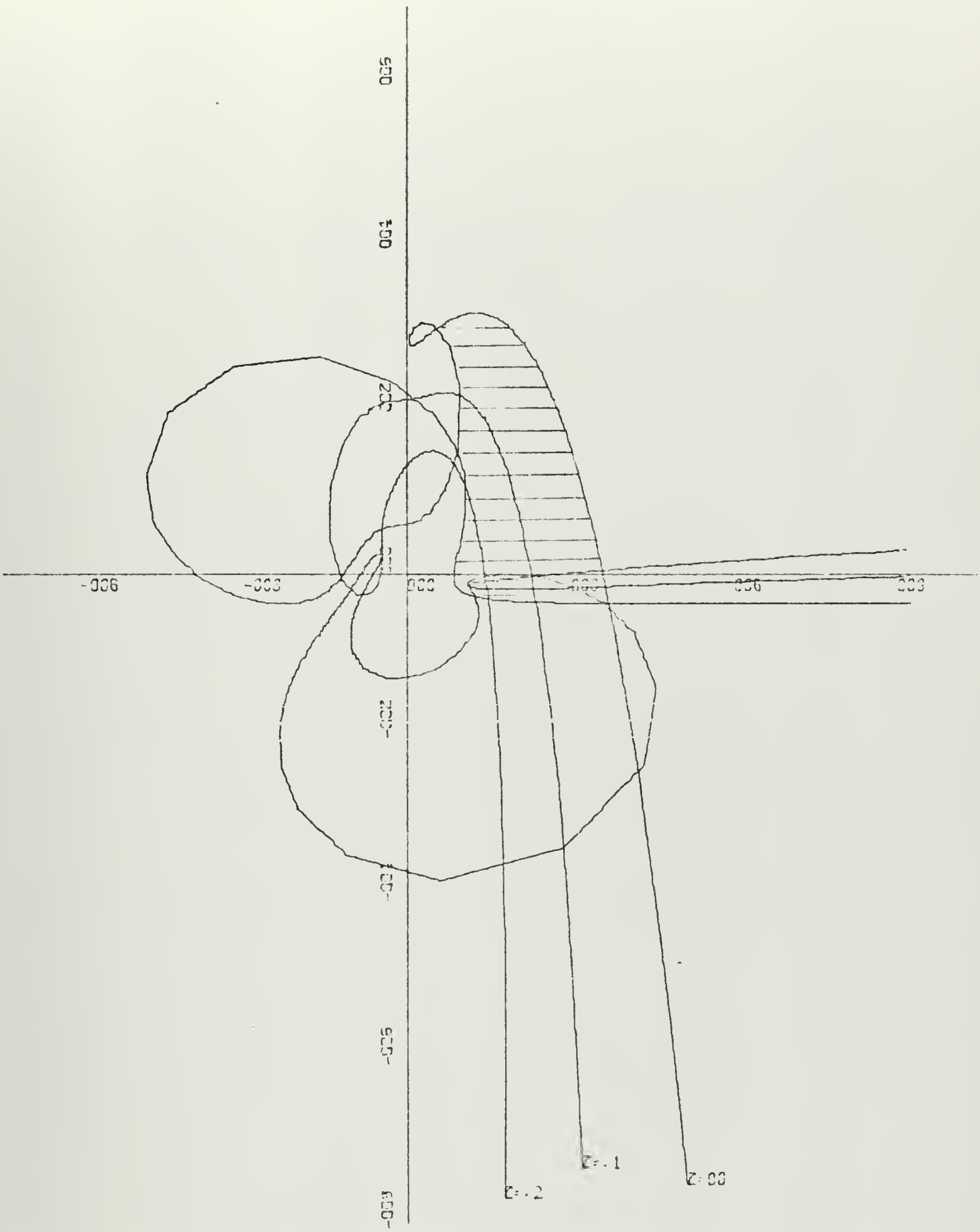
As in the previous example, the most acceptable transient response of the compensated system should be obtained by selecting values of α and β which lie within the area of intersection of the stable region and the area enclosed by the zeta = 0.2 curve. The transient response of the compensated system for values of α and β within the area of intersection under consideration is shown in Fig. 3.47. A comparison of Fig. 3.47 with Figs. 3.49 and 3.51, which represent the transient response of the system for other combinations of α and β, indicates that the assumption as stated was valid. Again verifying that a parameter plane analysis not only indicates whether a system can be stabilized by a selected method, but also what range of values of the parameters will result in a most favourable response of the compensated system.



Scale: x and y = 1.0/in

Figure 3.45

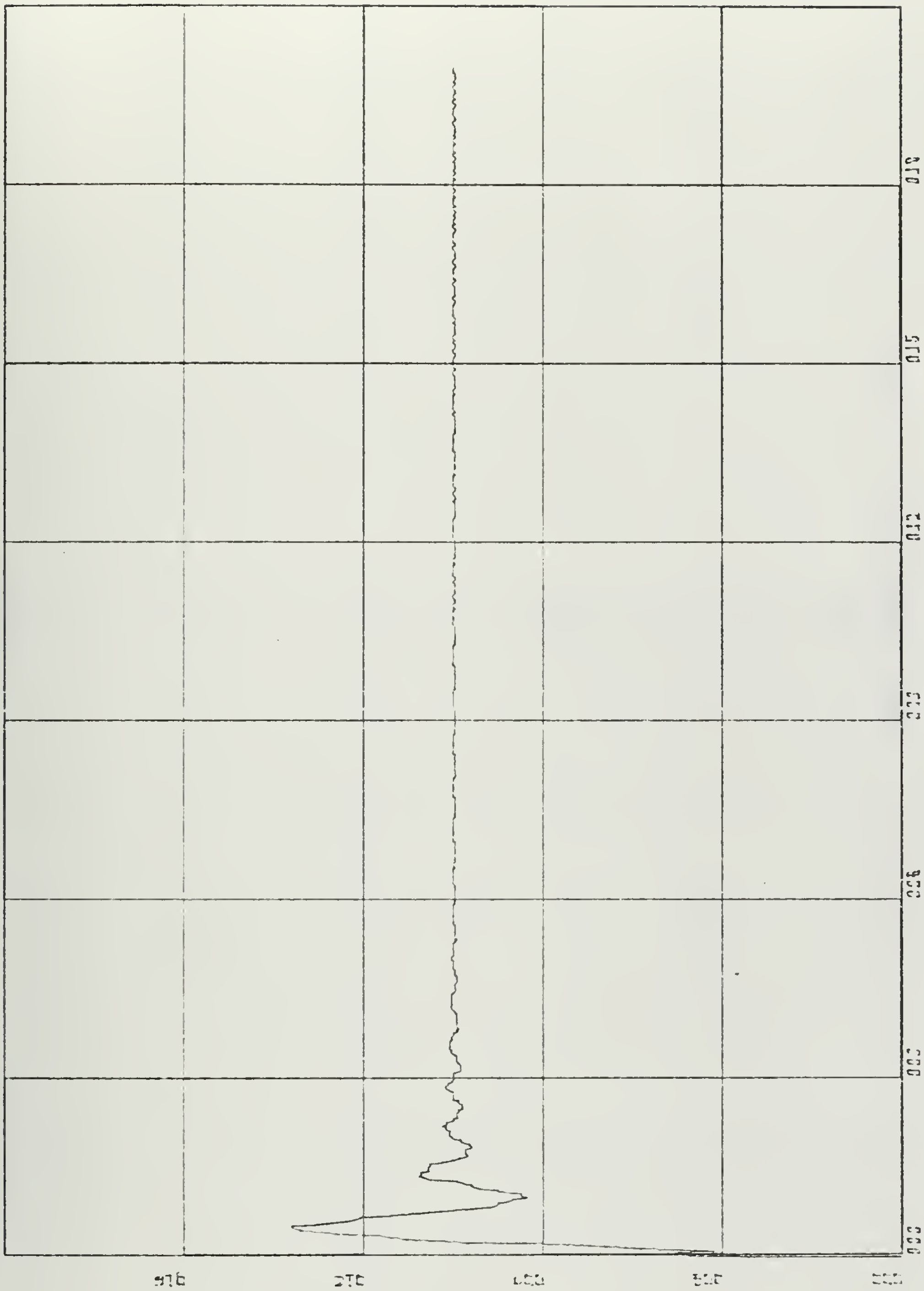
Root Locus of Uncompensated System with Two Doublets



Scale: α = 3.0/in, β = 2.0/in

Figure 3.46

Parameter Plane Curves for System with Two Doublets

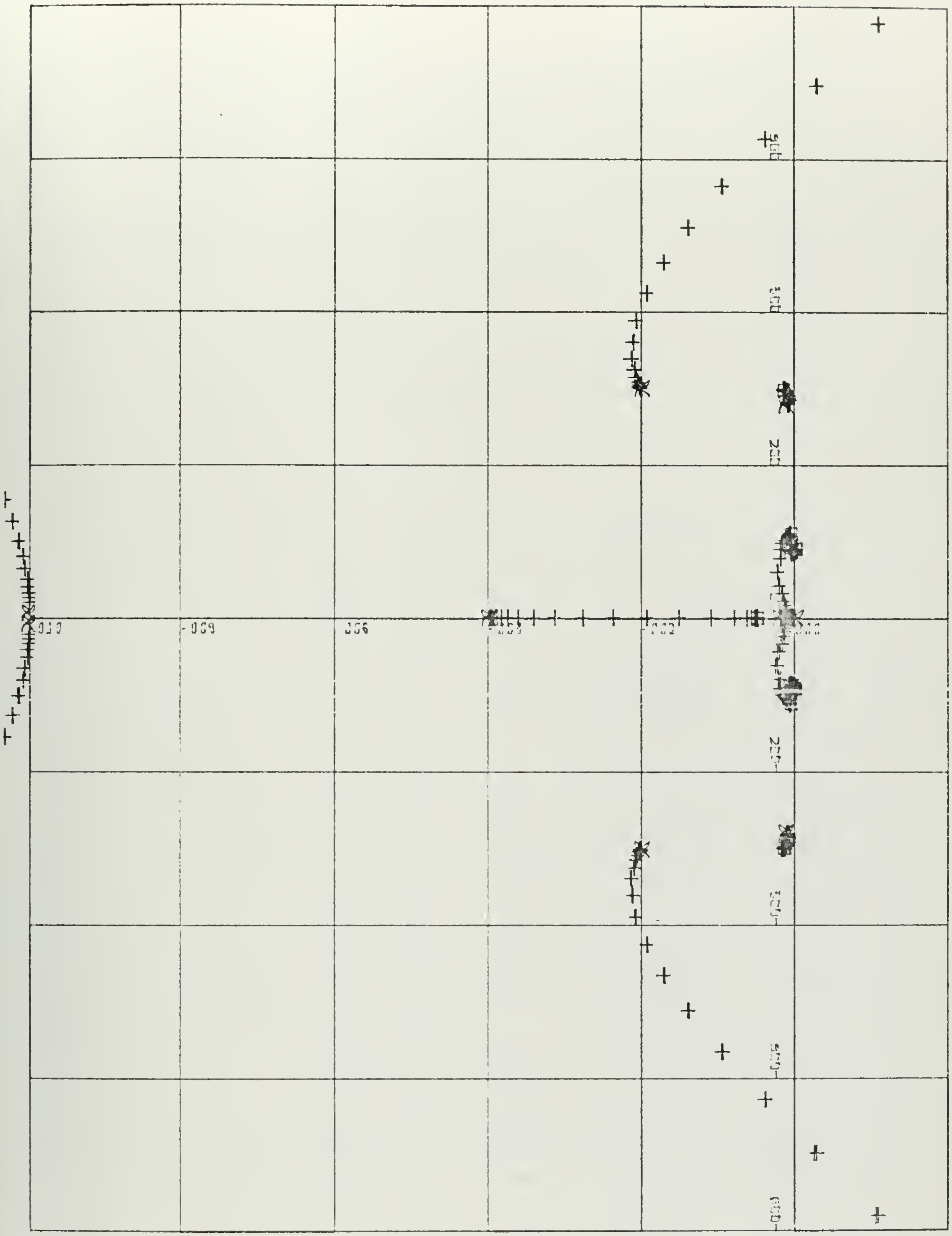


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.47

Transient Response of Compensated System with Two Doublets

$\alpha = 1.2, \quad \beta = 0.0$

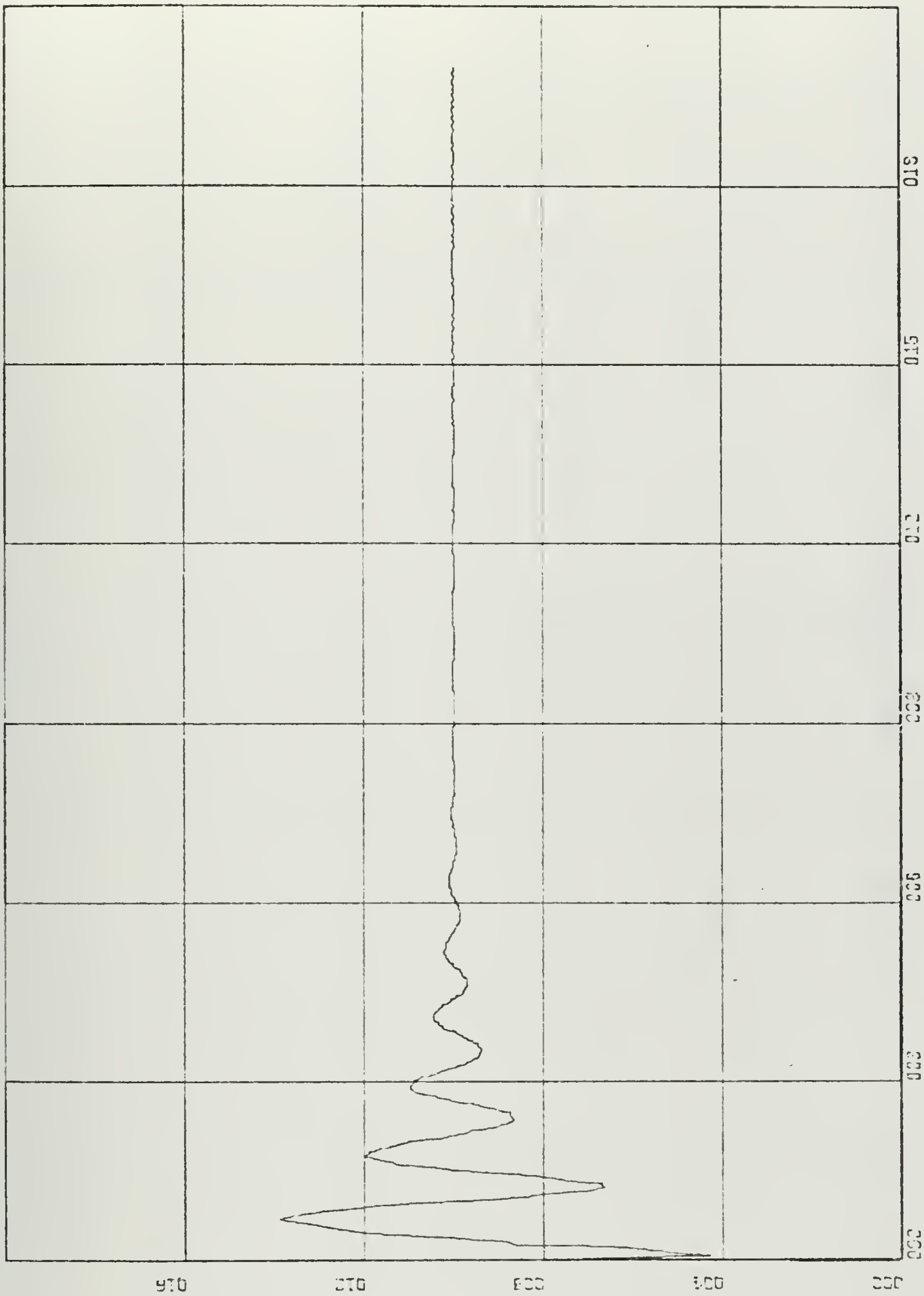


Scale: x and y = 2.0/in

Figure 3.48

Root Locus of Compensated System with Two Doublets

$\alpha = 1.2, \quad \beta = 0.0$

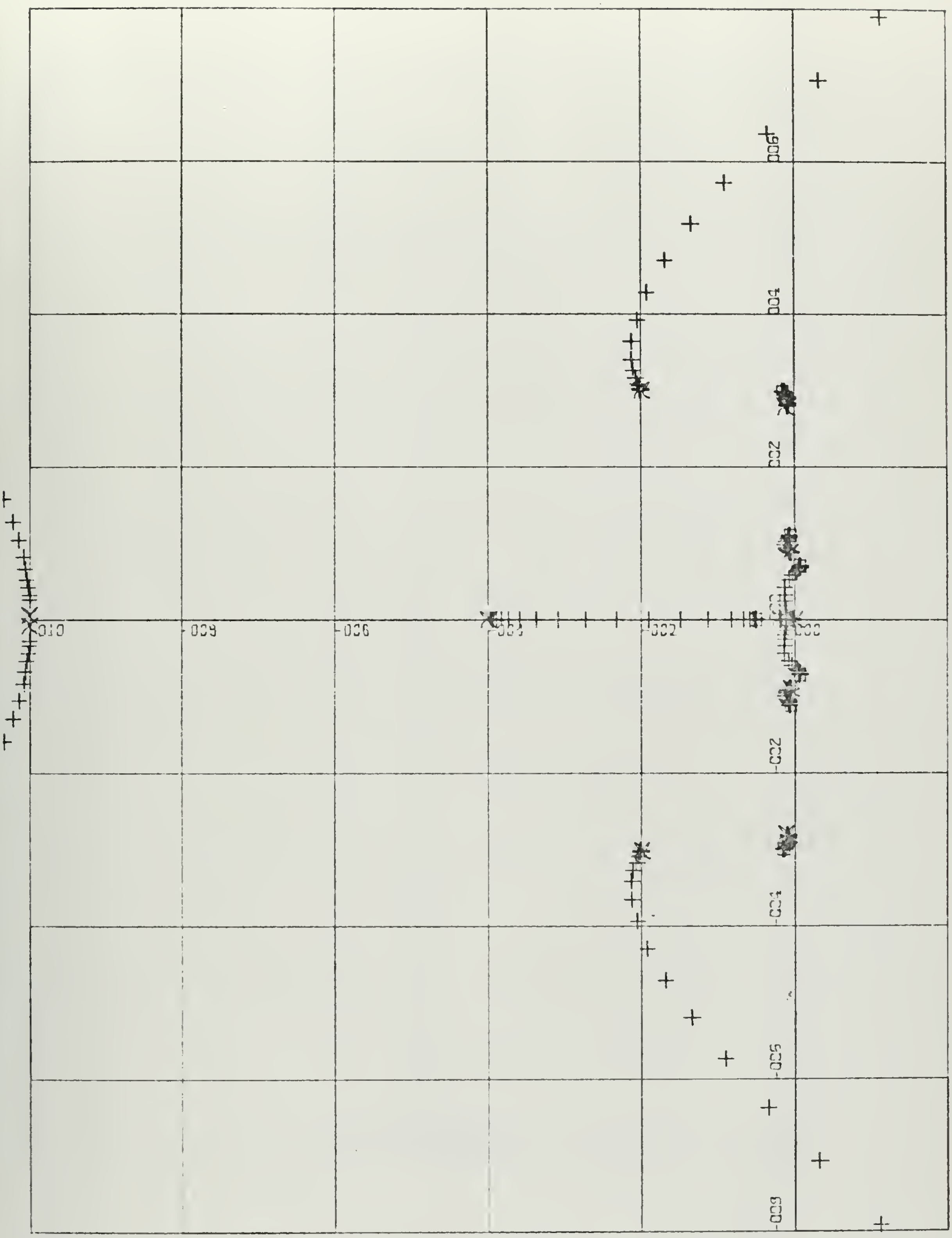


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.49

Transient Response of Compensated System with Two Doublets

$\alpha = 2.0$, $\beta = -0.15$

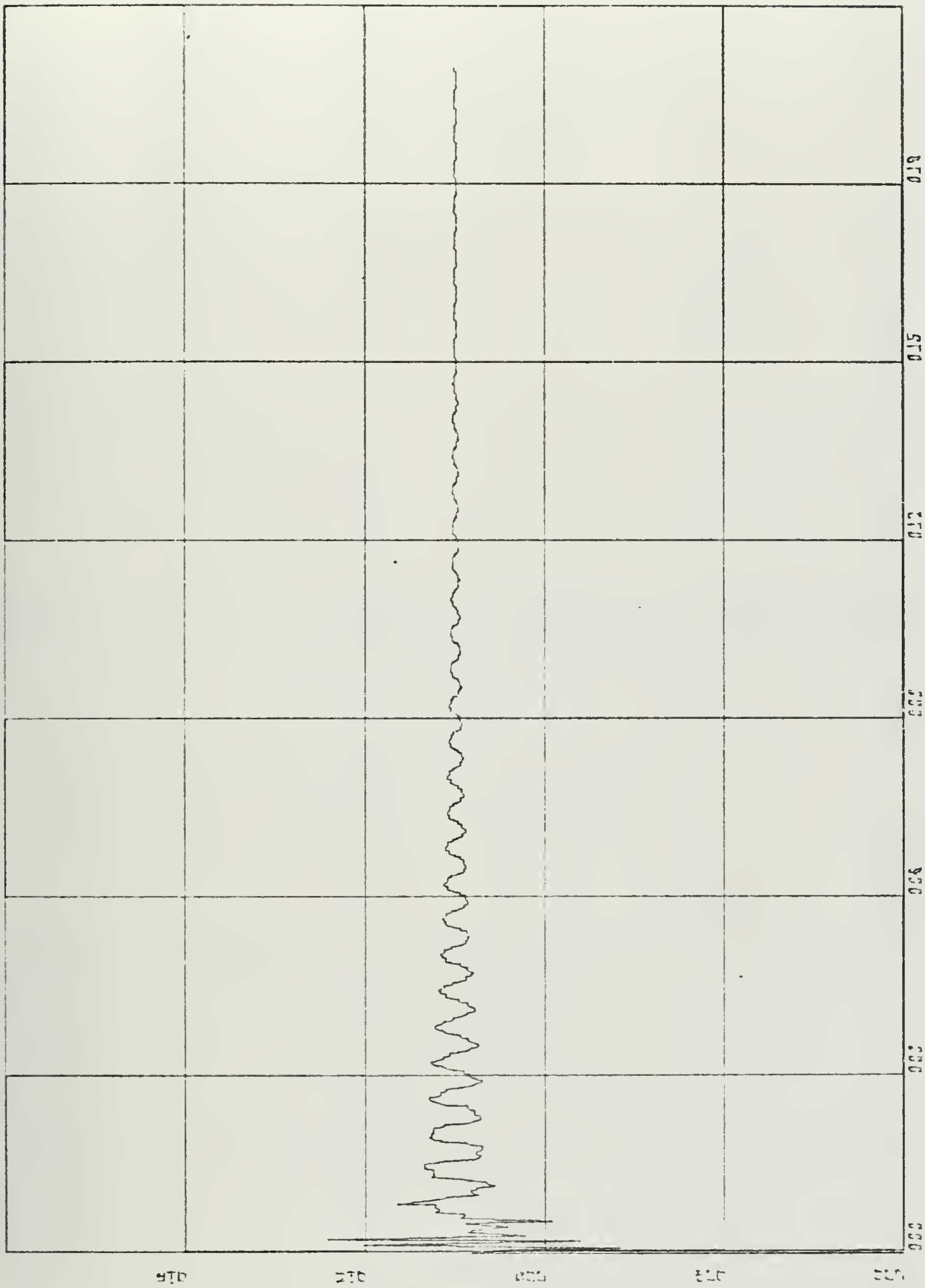


Scale: x and y = 2.0/in

Figure 3.50

Root Locus of Compensated System with Two Doublets

$\alpha = 2.0, \quad \beta = -0.15$

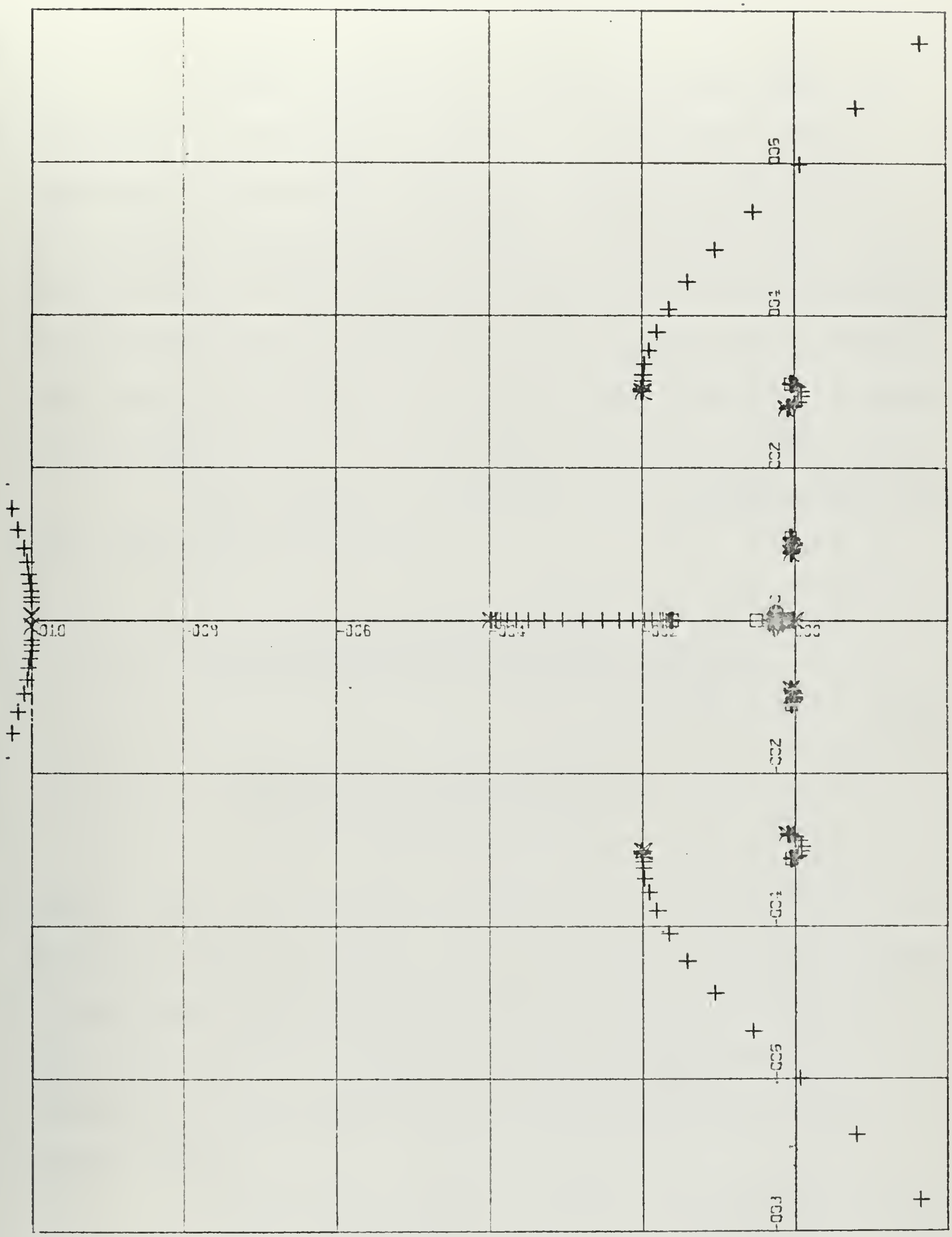


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.51

Transient Response of Compensated System with Two Doublets

$\alpha = 2.0, \quad \beta = 2.0$



Scale: x and y = 2.0/in

Figure 3.52

Root Locus of Compensated System with Two Doublets

$\alpha = 2.0, \quad \beta = 2.0$

c. A System with Three Doublets

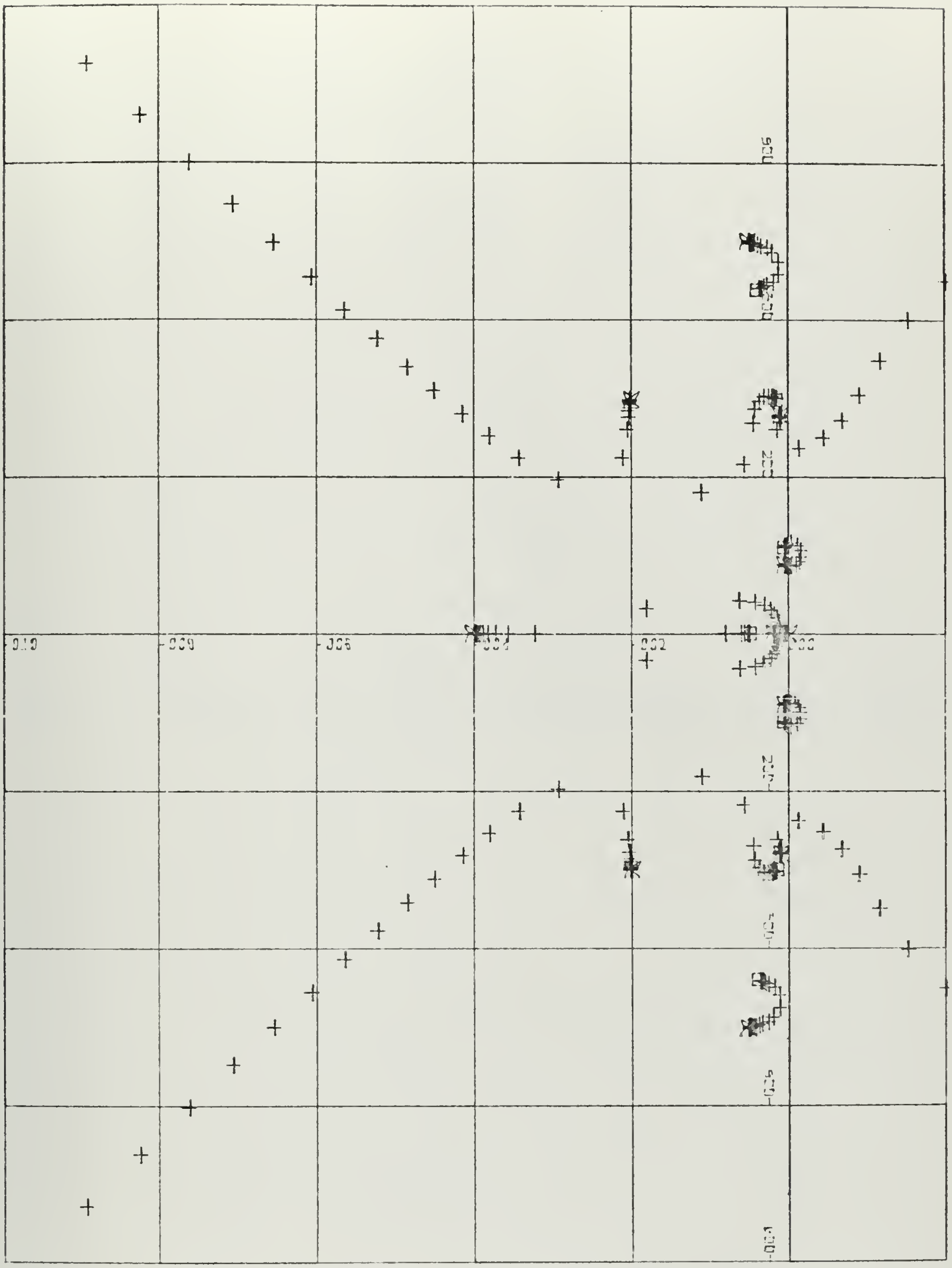
In the two previous examples of the effect of resonance - antiresonance doublets the undamped natural frequency of the complex poles was less than that of the accompanying complex zeros. This is, of course, not always the case and for that reason the undamped natural frequencies of the third doublet included in the system under investigation have been reversed, i.e., the undamped natural frequency of the complex zeros is less than that of the complex poles.

The transfer function of the uncompensated system with three doublets becomes:

$$G(s) = \frac{K(s+0.5)(s^2+0.12s+1.22)}{s(s+0.2)(s+4)(s^2+4s+13)(s^2+0.15+0.8)} \cdot \frac{(s^2+0.3s+9.0)(s^2+0.8s+19.5)}{(s^2+0.2s+7.85)(s^2+s+25.25)} \tag{3-19}$$

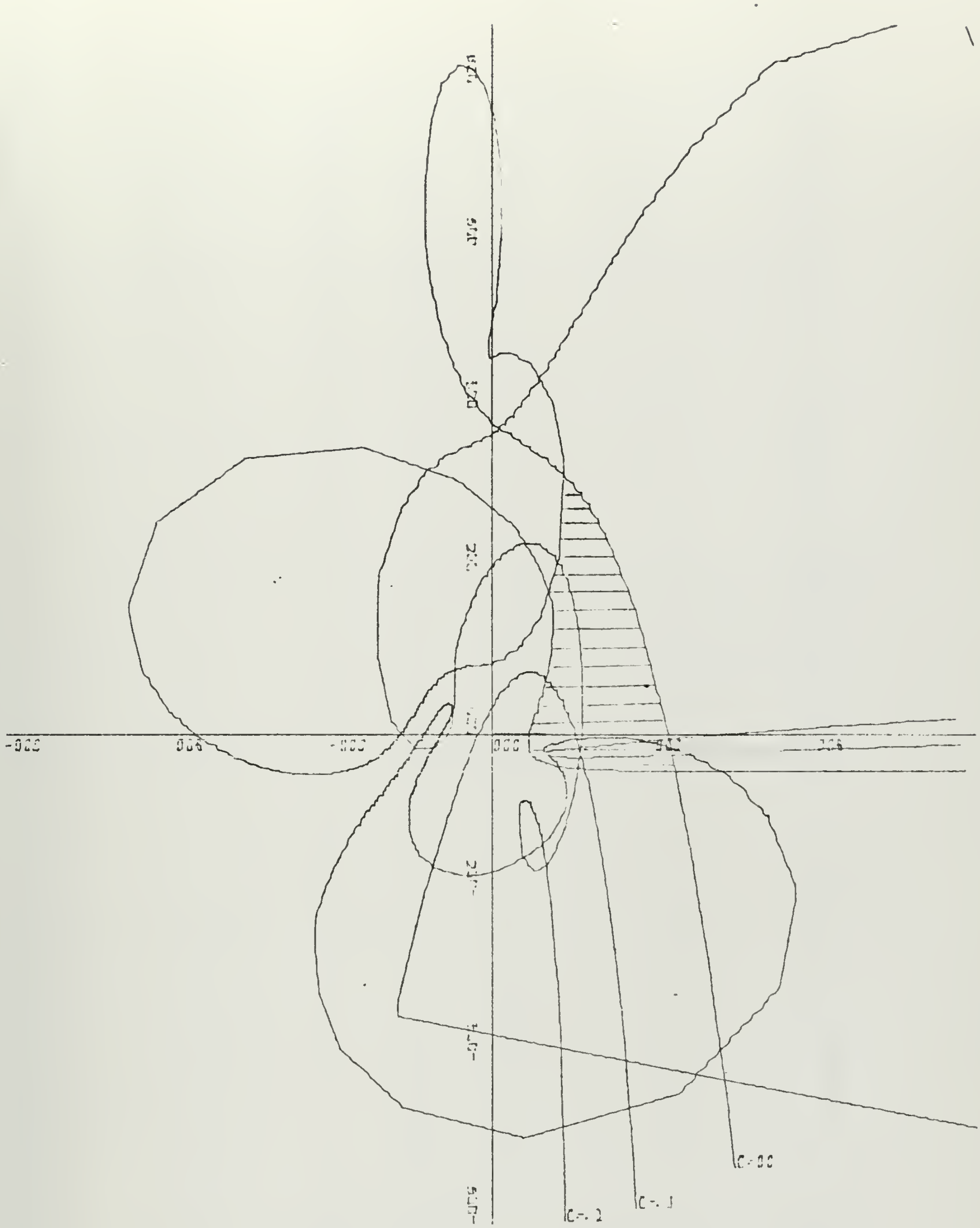
and the root locus of this system is shown in Fig. 3.53. It should be noted that Fig. 3.53 shows that the complex zeros of the third doublet are located closer to the origin than the complex poles indicating that the undamped natural frequency of the complex zeros is less than that of the complex poles.

Since the system, as before, is unstable because of the presence of the first doublet, as shown in Fig. 3.53, it is again reasonable to expect that a parameter plane analysis of the feasibility of stabilizing the system with



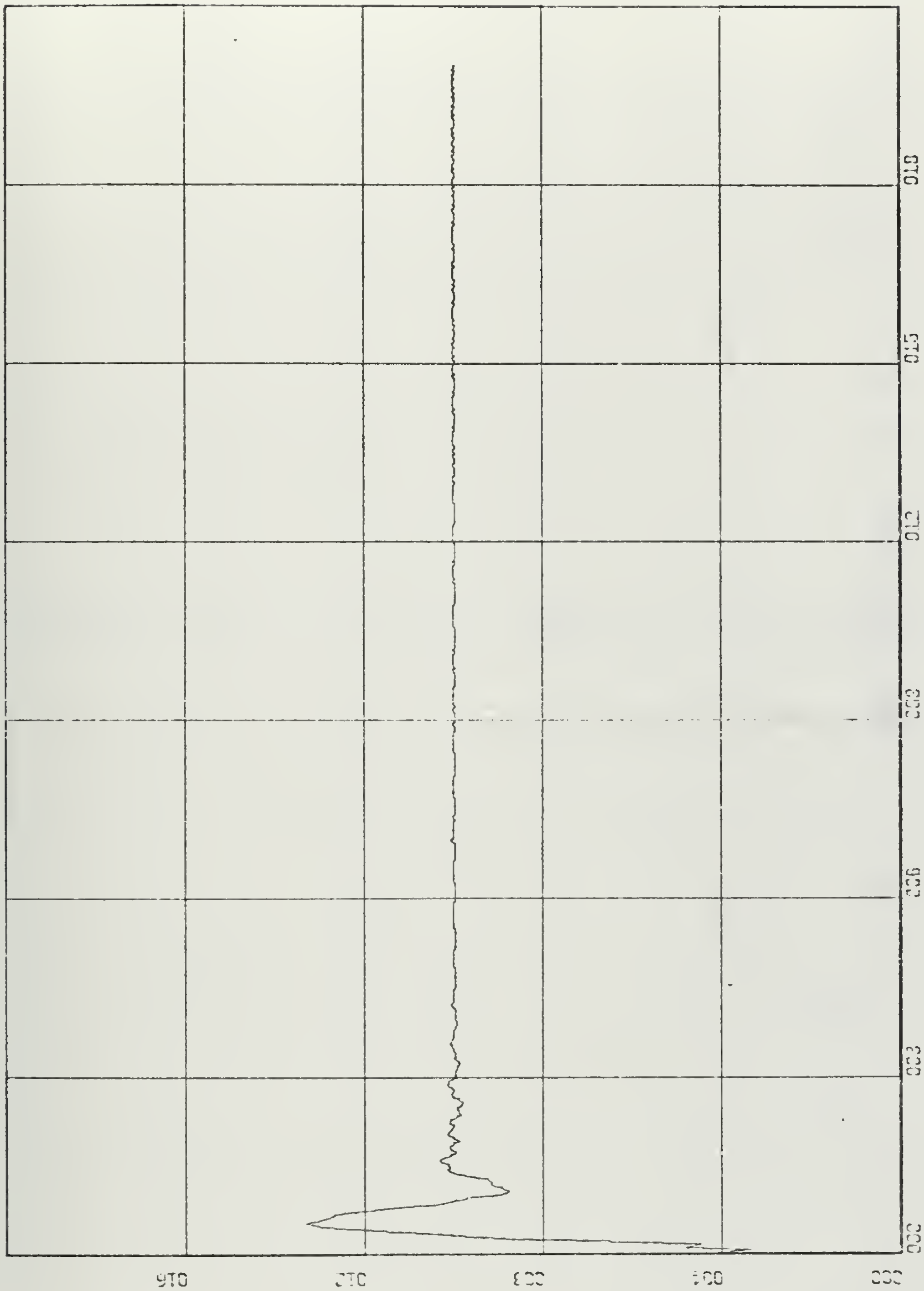
Scale: x and y = 2.0/in

Figure 3.53
Root Locus of Uncompensated System with Three Doublets



Scale: α = 3.0, β = 2.0

Figure 3.54
Parameter Plane Curves for System with Three Doublets

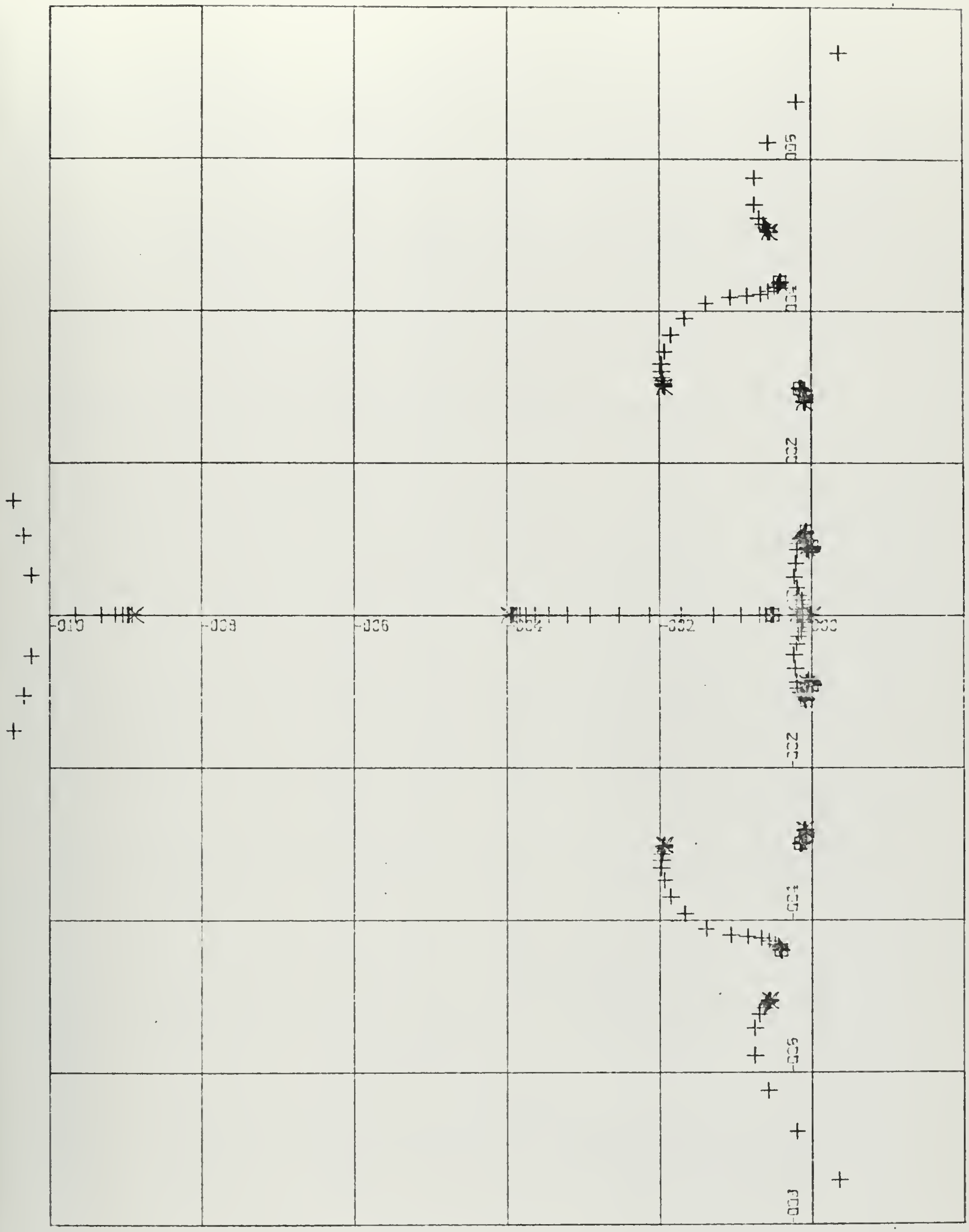


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.55

Transient Response of Compensated System with Three Doublets

$\alpha = 1.2$, $\beta = 0.0$

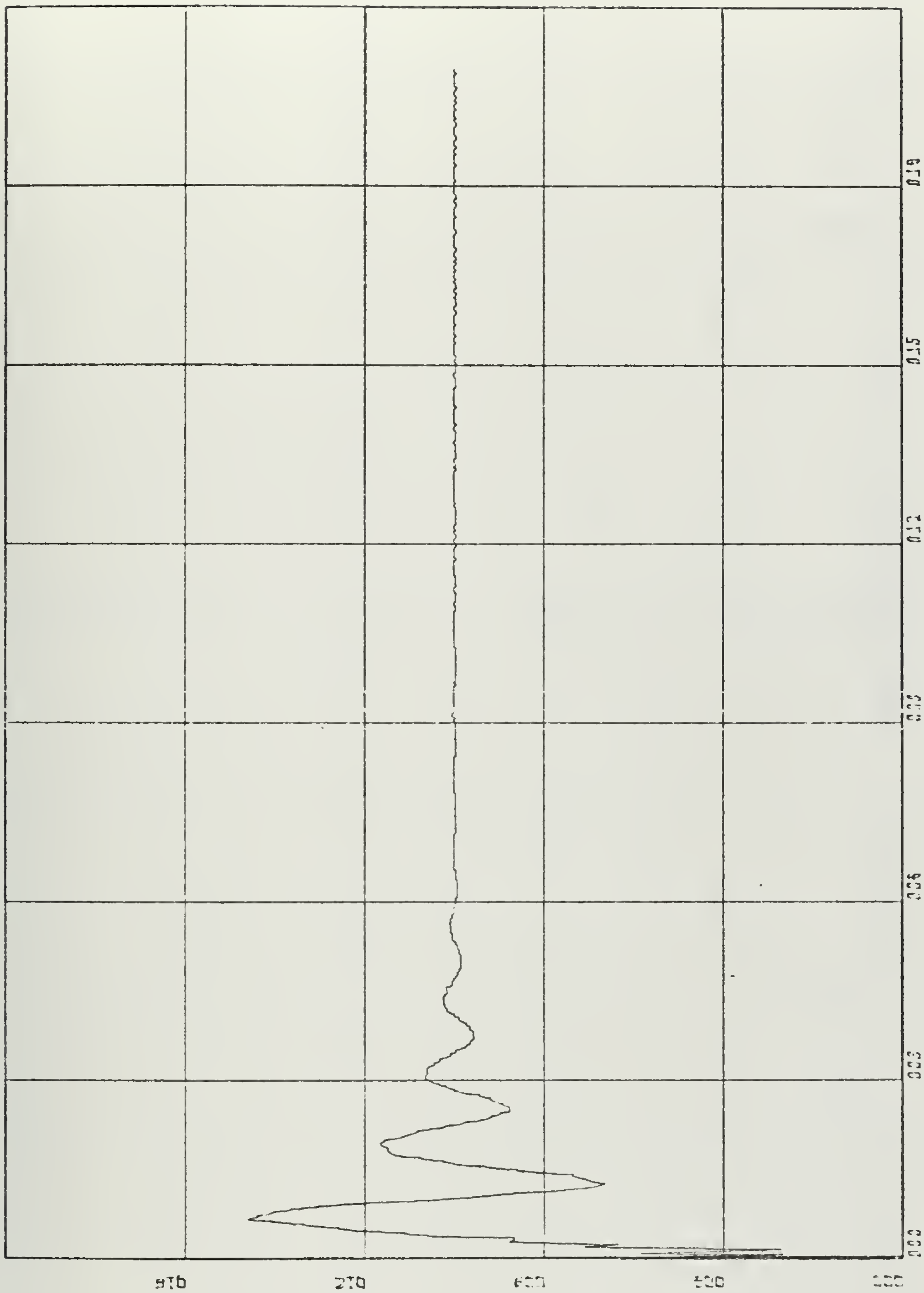


Scale: x and y = 2.0/in

Figure 3.56

Root Locus of Compensated System with Three Doublets

$\alpha = 1.2, \quad \beta = 0.0$

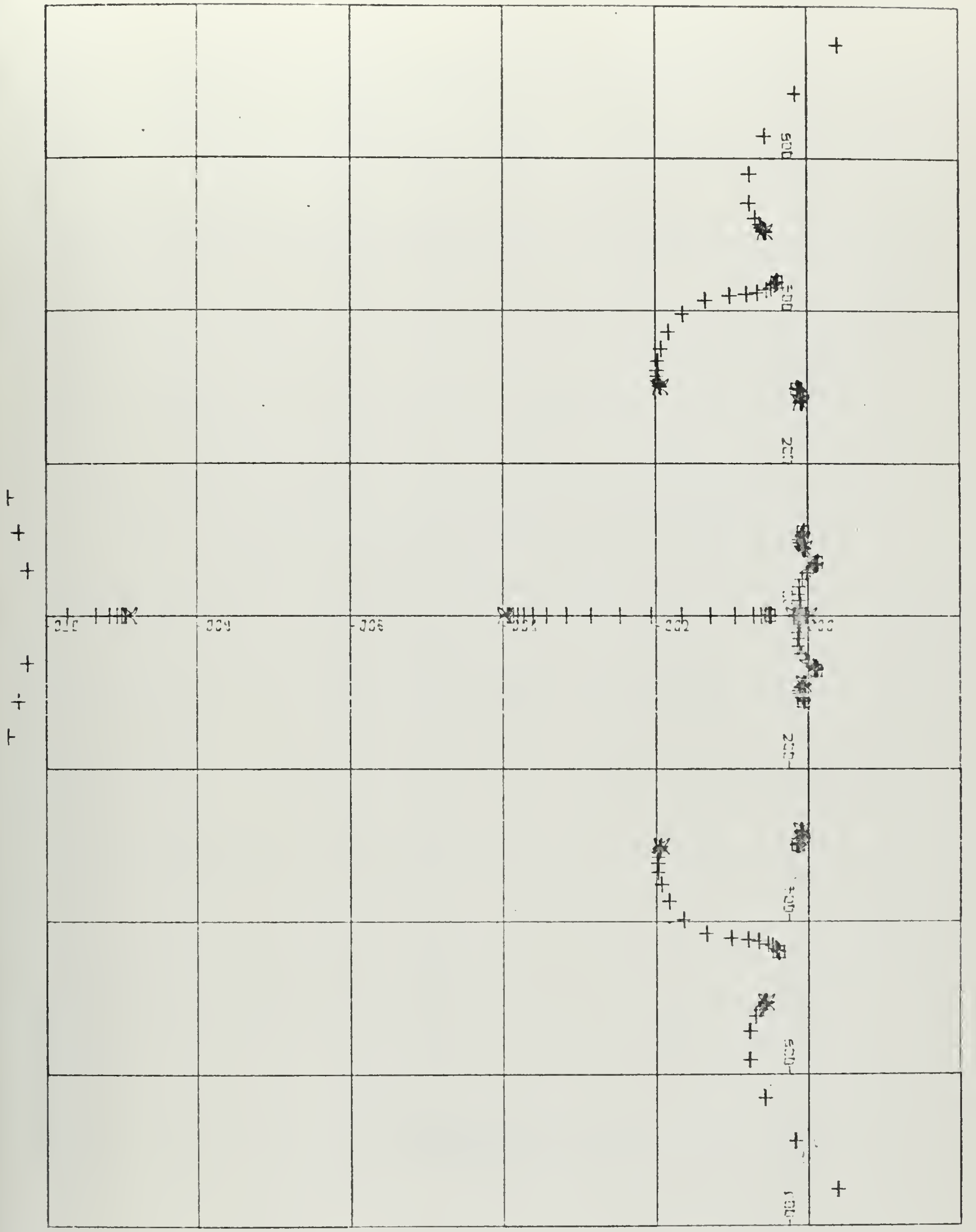


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.57

Transient Response of Compensated System with Three Doublets

$\alpha = 2.0$, $\beta = -0.2$

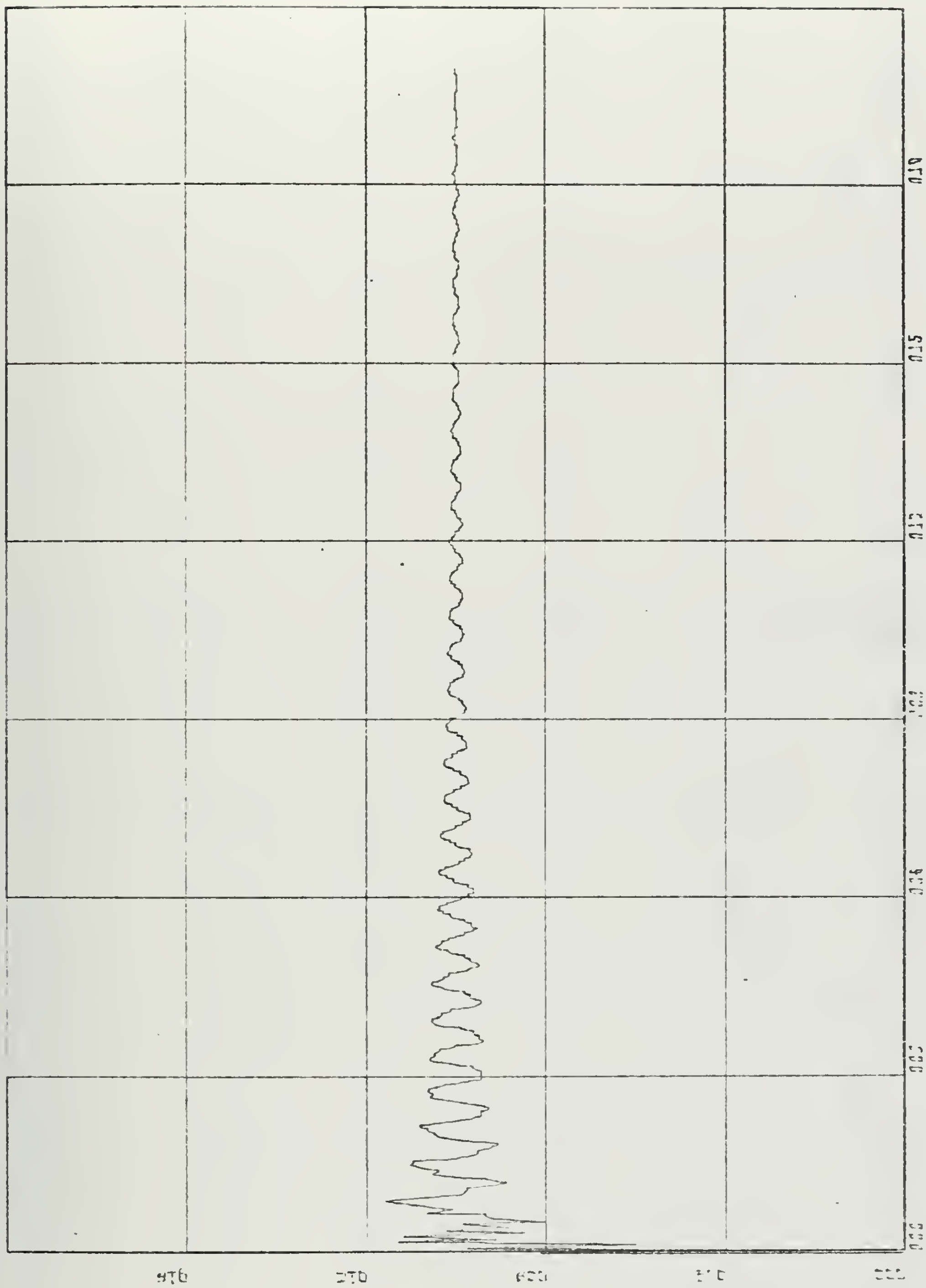


Scale: x and y = 2.0/in

Figure 3.58

Root Locus of Compensated System with Three Doublets

$\alpha = 2.0, \quad \beta = -0.2$

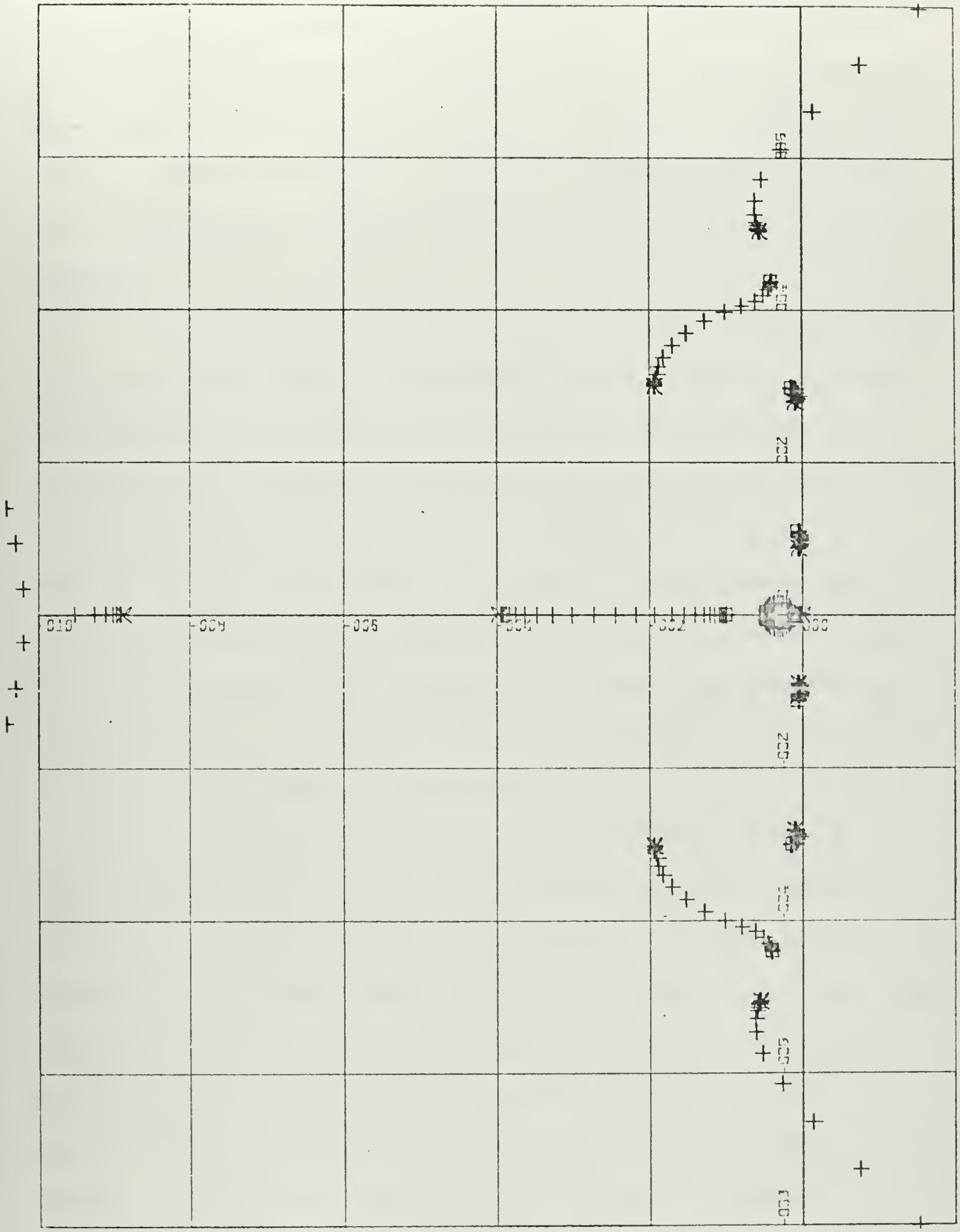


Scale: x = 30 sec/in, y = 0.4 units/in

Figure 3.59

Transient Response of Compensated System with Three Doublets

$\alpha = 2.0$, $\beta = 1.5$



Scale: x and y = 2.0/in

Figure 3.60

Root Locus of Compensated System with Three Doublets

$\alpha = 2.0, \quad \beta = 1.5$

one complex zero compensator of the form described by equation 3-13 should result in parameter plane curves resembling those shown in Figs. 3.38 and 3.46. This is, in fact, the case as illustrated by Fig. 3.54 which represents the parameter plane curves for the compensated system with three doublets.

Similar to the procedure followed in the previous two cases, three sets of values of α and β from within the stable region were chosen and the corresponding transient responses of the compensated systems are shown in Figs. 3.55, 3.57 and 3.59 together with their respective root loci in Figs. 3.56, 3.58 and 3.60. As expected, the most acceptable transient response was obtained for values of α and β within the area of intersection between the stable region and the area bounded by the zeta = 0.2 curve as shown in Fig. 3.55.

d. Discussion of Results

The analysis carried out in the last three sections have shown that a basic control system which is stable at a selected value of the error coefficient can become unstable at the same value of Kv if the effect of structural resonances is taken into account. It has also been shown that such an unstable system can be stabilized by the use of a complex zero compensator if the location of the pair of complex zeros or real zeros is such that the root loci of the compensated system are adjusted so that none of the roots of the closed loop system at the specified gain are located in the right half of the 's' plane. The problem of finding the correct location of the zeros of the compensator to achieve

stability of the system is solved by the use of a parameter plane analysis where the two variable parameters are selected to be functions of the location of the zeros of the compensator. A parameter plane analysis not only indicates the proper location of the zeros of the compensator to achieve stability by the use of the zeta = 0.0 curve, but also gives an indication of where the zeros should be located to assure a desired type of transient response by the use of constant zeta curves of values other than zero.

The optimum transient response of a system affected by structural resonances and stabilized by the use of a cascade complex zero compensator may not approach the desired response achieved by the system without structural resonances. It is obvious that the responses shown in Figs. 3.39, 3.47 and 3.55 are not as good as the one shown in Fig. 3.36, but under the given conditions they are approximately the best that can be obtained from the standpoint of maximum overshoot and settling time. If an area of intersection between the stable region and an enclosed area bounded by a constant zeta curve of value greater than 0.2 had been indicated, a more favourable transient response could have been possible. The parameter plane curves shown in Figs. 3.38, 3.46 and 3.54 indicate, however, that this is not the case in the presented examples and that under the given conditions and for the selected compensator the optimum has been reached. The value of a parameter plane analysis lies, therefore, in the immediate availability of the location

of the zeros of the compensator to achieve stability of the system and in the information available from which a first approximation of the obtainable transient response can be formed.

IV. CONCLUSIONS

Control systems affected by mechanical or structural resonances contain at least one pair of complex poles in the corresponding transfer functions. The presence of the complex poles causes resonant peaks to occur in the open-loop frequency response of the system which may produce instability in the closed-loop system. The use of complex zero compensators to stabilize such systems has been widely accepted particularly when complete cancellation of the complex pole by the complex zero of the compensator can be achieved.

Exact cancellation by one pair of complex zeros is, of course, impossible to achieve if the frequency of the mechanical or structural resonance varies during the operation of the system, or if resonances at two or more different frequencies occur. Such a system can, however, be stabilized by a cascade compensator containing one pair of complex or two real zeros if the choice of zeros is such that they modify the root locus of the system in a manner which will ensure that all roots of the closed-loop system remain in the left half of the 's' plane. A parameter plane analysis, using functions of the location of the complex zeros as the two variables, provides an effective method of determining the correct location of the zeros of the compensator to ensure stability. At least one other important system criteria, such as constant error coefficient, may be imbedded in the

analysis to ensure that the solution obtained will satisfy the specified criteria as well.

A parameter plane analysis may be used not only as a method to determine the correct location of compensating zeros to ensure stability, but also as an aid in the design of the compensator to achieve a desired system performance. The exact response of the compensated system can not be predicted from a parameter plane analysis, but an approximation based on second order system theory can be obtained if constant zeta curves of values other than zero are plotted on the parameter plane.

It is felt that a parameter analysis using cascade complex zero compensators, as has been illustrated in this investigation, can provide considerable aid to the practical designer faced with the problem of a system affected by mechanical or structural resonances.

The examples presented in this investigation and the conclusions drawn from them by no means exhaust the study of the applicability of parameter plane analysis in the design of complex zero compensators. The following questions are presented as suggestions for possible future investigations.

1. What is effect of the location of the poles of the compensator, i.e., does their location seriously affect the size of the stable region on the parameter plane?
2. Can a system containing several doublets be stabilized with one pair of complex zeros if

the root locus geometry is such that the root loci of two doublets enter the right-half plane and assuming that, at sufficiently high gain, the roots of the lower frequency doublet reenter the left-half plane, thus causing instability of the system due to the second doublet?

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13. ABSTRACT

Control systems affected by mechanical or structural resonances contain at least one pair of complex poles in their corresponding transfer functions. The effect of the complex poles is to introduce resonance peaks which may cause instability in the closed-loop system. Such systems may be stabilized by the use of cascade compensators containing one pair of complex zeros.

The use of a parameter plane analysis, using a function of the location of the complex zeros as the two variables, in determining the correct location of the zeros of the compensator to guarantee stability and to ensure a desired transient response in the compensated system is illustrated.

As a result of the investigation of a number of systems affected by resonance phenomena using parameter plane techniques, design criteria for cascade complex zero compensators, based upon the interpretation of the 'stable region' of the parameter plane, are presented.

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